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AUTOMATION AND REMOTE CONTROL

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THE ABSOLUTE STABILITIES OF CERTAIN CONTROLLED SYSTEMS

Yu.S. Sobolev

(Leningrad)

A problem [1, 2] of comparison in the theory of absolute stability for automatic control systems has been solved. Some examples are given.

1. A problem, which is formulated in a more general form below, has been posed in [1, 2].

We have two sets of differential equations

$$\dot{x}_i = f_i(x_1, \ldots, x_n) \quad (i = 1, \ldots, n-m),$$
$$\dot{x}_k = f_k(x_1, \ldots, x_n) + b_k\varphi_k[\sigma_k(x_1, \ldots, x_n)] \quad (k = n-m+1, \ldots, n), \tag{1}$$

$$\dot{x}_i = f_i(x_1, \ldots, x_n) \quad (i = 1, \ldots, n-m),$$
$$\dot{x}_k = f_k(x_1, \ldots, x_n) + h_k b_k \sigma_k(x_1, \ldots, x_n) \quad (k = n-m+1, \ldots, n). \tag{2}$$

Let these sets satisfy these conditions:

a) solutions to (1) and (2) exist and are unique in the region of every point throughout the space of variables $x_1, \ldots, x_n$;

b) the point $(0, \ldots, 0)$ is a unique state of equilibrium for both systems;

c) the function φ_k belongs to class A [1]:

$$\varphi_k(\sigma_k)\,\sigma_k > 0, \ \sigma_k > 0, \text{ when } \sigma_k \neq 0 \text{ and } \varphi_k(0) = 0 \quad (k = n-m+1, \ldots, n);$$

d) $\sigma_k(x_1, \ldots, x_n) = 0$ only when

$$x_1 = x_2 = \ldots = x_n = 0 \qquad (k = n-m+1, \ldots, n);$$

e) b_k and h_k are real numbers, and $h_k \in (0, +\infty)$;

f) the square of the "distance" of (2)'s image point from the origin dies away strictly monotonically for any $h_k \in (0, +\infty)$, and when $t \to +\infty$.*

Prove that the square of the "distance" of (1)'s image point from the origin also dies away strictly monotonically.

The proof is as follows. We multiply the i-th (k-th) equation in (1) by $r\dfrac{\partial r}{\partial x_i}\left(r\dfrac{\partial r}{\partial x_k}\right)$ and sum the resulting equations from 1 to n:

*By "distance" (henceforth denoted by r) we mean a positively defined continuously differentiable function of n variables.

$$\frac{1}{2}\frac{d(r^2)}{dt}=\sum_{l=1}^{n} r\frac{\partial r}{\partial x_l}f_l(x_1,\ldots,x_n)+\sum_{k=n-m+1}^{n} r\frac{\partial r}{\partial x_k}b_k\varphi_k[\sigma_k(x_1,\ldots,x_n)]. \tag{3}$$

We do the same with (2):

$$\frac{1}{2}\frac{d(r^2)}{dt}=\sum_{l=1}^{n} r\frac{\partial r}{\partial x_l}f_l(x_1,\ldots,x_n)+\sum_{k=n-m+1}^{n} r\frac{\partial r}{\partial x_k}h_kb_k\sigma_k(x_1,\ldots,x_n). \tag{4}$$

Consider (4). Condition "f" implies that the expression on the right is negative for any values for which $h_k \in (0, +\infty)$ $(k = n-m+1, \ldots, n)$ throughout the space of the variables $x_1, \ldots, x_n$.

We use this to find the signs of the terms on the right in (4). We assume that all the h_k $(k = n-m+1, \ldots, n)$ are sufficiently small, and so we have that

$$\sum_{l=1}^{n} r\frac{\partial r}{\partial x_l}f_l(x_1,\ldots,x_n)\leqslant 0\,^{*} \tag{5}$$

for any $x_1, \ldots, x_n$.

Further, we make one of the h_s, e.g., h_s, large, and all the others h_p $(p = n-m+1, \ldots, s-1, s+1, \ldots, n)$ small; we have

$$r\frac{\partial r}{\partial x_s}b_s\sigma_s(x_1,\ldots,x_n)<0. \tag{6}$$

Now s is arbitrary, and so we can say that

$$r\frac{\partial r}{\partial x_k}b_k\sigma_k(x_1,\ldots,x_n)<0 \quad (k=n-m+1,\ldots,n) \tag{7}$$

for any $x_1, \ldots, x_n$.

Consider now the product

$$r^2\left(\frac{\partial r}{\partial x_k}\right)^2 b_k^2\sigma_k(x_1,\ldots,x_n)\varphi_k[\sigma_k(x_1,\ldots,x_n)]. \tag{8}$$

Condition "c" shows that this is always positive.

Hence the sign of $r\frac{\partial r}{\partial x_k}b_k\varphi_k[\sigma_k(x_1,\ldots,x_n)]$ throughout the space of $x_1, \ldots, x_n$ is the same as that of $r\frac{\partial r}{\partial x_k}b_k\sigma_k(x_1,\ldots,x_n)$. From (6) we have that

*In fact, for any set of fixed values $x_1^*, \ldots, x_n^*$ we can always either put down a set of values h_k^* $(k = n-m+1, \ldots, n)$ such that

$$\left|\sum_{k=n-m+1}^{n} h_k^* r\frac{\partial r}{\partial x_k}b_k\sigma_k(x_1^*,\ldots,x_n^*)\right|<\left|\sum_{l=1}^{n} r\frac{\partial r}{\partial x_l}f_l(x_1,\ldots,x_n)\right|,$$

or can readily show that

$$\sum_{l=1}^{n} r\frac{\partial r}{\partial x_l}f_l(x_1,\ldots,x_n)=0.$$

$$r\frac{\partial r}{\partial x_k}b_k\varphi_k[\sigma_k(x_1,\ldots,x_n)]<0 \qquad (k=n-m+1,\ldots,n) \tag{9}$$

for any $x_1, \ldots, x_n$.

Now from (3), (5) and (9) we get

$$\frac{1}{2}\frac{d(r^2)}{dt}=\sum_{l=1}^{n} r\frac{\partial r}{\partial x_l}f_l(x_1,\ldots,x_n)+\sum_{k=n-m+1}^{n} r\frac{\partial r}{\partial x_k}b_k\varphi_k[\sigma_k(x_1,\ldots,x_n)]<0.$$

Hence the square of the distance of (1)'s image point from the origin dies away strictly monotonically throughout the space of the variables $x_1, \ldots, x_n$.

Note. In the space $x_1, \ldots, x_n$ we consider a set of differential equations

$$\dot{x}_i=\sum_{j=1}^{n}a_{ij}x_j \quad (i=1,\ldots,n), \quad a_{ij}=\text{const}, \tag{10}$$

$$\|A\|=\begin{Vmatrix} a_{11}\ldots a_{1n} \\ a_{1n}\ldots a_{nn}\end{Vmatrix}, \tag{11}$$

and a form derived from (10) by multiplying the i-th equation by $r\frac{\partial r}{\partial x_i}$, and summing the resulting expressions:

$$\frac{1}{2}\frac{d(r^2)}{dt}=\sum_{i,j=1}^{n,n}a_{ij}r\frac{\partial r}{\partial x_i}x_j. \tag{12}$$

where $r^2=\sum_{i,j=1}^{n,n}b_{ij}x_ix_j$ is a definitely positive quadratic form. The coefficients in this form can be so chosen that the complete derivative of r^2, with respect to t, will, from (10), be negative when $t > 0$ if, and only if, $\|A\|$ has eigenvalues with negative real parts.

Now we have assumed that r^2 is an arbitrary function; hence we use the above proof and note to justify our assuming that there is a function r^2 whose complete derivative is, on account of (1), negatively defined, by virtue of the Rous-Gurvits conditions.

2. Consider some examples. We assume that the local conditions of existence and uniqueness for the solution are complied with, and that the point $(0, \ldots, 0)$ is a unique state of equilibrium.

Example 1. Find the absolute stability bound for a control system with one controller*:

$$\begin{aligned} T_a^2\ddot{y}_1+U\dot{y}_1&=-Ny_2, \\ T_1^2\ddot{y}_2+W\dot{y}_2&=f(\sigma), \end{aligned} \tag{13}$$

where $\sigma = ay_1 + E\dot{y}_1 + G^2\ddot{y}_1 - ry_2$, and $f(\sigma)$ is of class A [1].

The proof above shows that the absolute stability region for (13) is the same as the asymptotic stability region for the system

$$\begin{aligned} T_a^2\ddot{y}_1+U\dot{y}_1&=-Ny_2, \\ T_{12}\ddot{y}_2+W\dot{y}_2&=h(ay_1+E\dot{y}_1+G^2\ddot{y}_1-ry_2), \end{aligned} \tag{14}$$

where $h \in (0, +\infty)$.

We use a method given in [1] to find the bounds to the asymptotic stability region.

*Such systems have been dealt with in [1, 4-6] and elsewhere.

The characteristic equation to (14) is of the form

$$T_i^2T_a^2\lambda^4 + (T_i^2U + WT_a^2)\lambda^3 + (WU + hrT_a^2 + hNG^2)\lambda^2 + (rUh + NEh)\lambda + hNa = 0.$$

If U, W, N, E, a, r > 0, the following inequality is the sole important one:

$$(hrU + hNE)[(T_i^2U + WT_a^2)(WU + rhT_a^2 + hNG^2) - T_i^2T_a^2(hrU + hNE)] - \\ - (T_i^2U + WT_a^2)^2hNa > 0.$$

The equation for the bounds to the asymptotic stability region for (14) is

$$h^3[(T_i^2U + WT_a^2)(rT_a^2 + NG^2)(rU + NE) - (rU + NE)^2T_i^2T_a^2] + \\ + h[(T_i^2U + WT_a^2)(rU + NE)WU - (T_i^2U + WT_a^2)^2Na] = 0.$$

The envelope to the family of curves described by h is given by

$$T_a^2W(NG^2 + rT_a^2) = NT_i^2(ET_a^2 - G^2U), \\ (rU + NE)WU = (T_i^2U + WT_a^2)Na. \tag{15}$$

Hence (15) is the bound to the absolute stability region for (13).

Example 2. Find the absolute stability bound for a control system with one controller [1]:

$$T_a^2\ddot{y}_1 + U\dot{y}_1 + ky_1 = -Ny_2, \\ W\dot{y}_2 + Sy_2 = f(\sigma), \tag{16}$$

where $f(\sigma)$ is a function of class A, and $\sigma = ay_1 + E\dot{y}_1 + G^2\ddot{y}_1 - ry_2$.

As before, the absolute stability region for (16) is the same as the asymptotic stability region for the system

$$T_a^2\ddot{y}_1 + U\dot{y}_1 + k\dot{y}_1 = -Ny_2, \\ W\dot{y}_2 + Sy_2 = h(ay_1 + E\ddot{y}_1 + G^2\dot{y}_1 - ry_2), \tag{17}$$

where $h \in (0, +\infty)$.

The characteristic equation to (17) is of the form

$$WT_a^2\lambda^3 + [WU + T_a^2(S + hr) + NG^2h]\lambda^2 + [Wk + U(S + hr) + \\ + NhE]\lambda + [Nah + k(S + hr)] = 0.$$

If U, W, S, N, E, k, a, r > 0, the following inequality is the sole important one:

$$[WU + T_a^2(S + hr) + NG^2h][Wk + U(S + hr) + NhE] - [Nah + k(S + hr)]WT_a^2 > 0.$$

The equation for the bounds to the asymptotic stability region for (17) is

$$h^3[(T_a^2r + NG^2)(rU + NE)] + h[(WU + T_a^2S)(rU + NE) + (kW + US)(T_a^2r + NG^2) - \\ - WT_a^2(Na + kr)] + (WU + T_a^2S)(Wk + US) - WT_a^2kS = 0.$$

The envelope to the family of curves described by h is given by

$$[(rU + NE)(WU + T_a^2S) + SU(rT_a^2 + NG^2) + NW(G^2k - T_a^2a)]^2 = \\ = 4(rT_a^2 + NG^2)(rU + EN)[SU(WU + T_a^2S) + kUW^2]. \tag{18}$$

The last equation is the bound to the absolute stability region for (16).*

*Other examples of such nonlinear control systems are dealt with in [1] and [5].

Example 3. Consider the system*

$$\begin{aligned} \dot{x} &= a_{11} f_1(x) + a_{12} y, \\ \dot{y} &= a_{21} x + a_{22} f_2(y), \end{aligned} \tag{19}$$

where $f_1(x)$ and $f_2(y)$ are functions of class A.

The absolute stability region for (19) is the same as the asymptotic stability region for the system:

$$\begin{aligned} \dot{x} &= a_{11} h_1 x + a_{12} y, \\ \dot{y} &= a_{21} x + a_{22} h_2 y, \end{aligned} \tag{20}$$

where $h_1 \in (0, +\infty)$, $h_2 \in (0, +\infty)$.

The characteristic equation to (20) is of the form

$$\lambda^2 - (h_1 a_{11} + h_2 a_{22})\lambda + (h_1 h_2 a_{11} a_{22} - a_{21} a_{12}) = 0.$$

The zero solution to (20) is asymptotically stable if

$$a_{11} < 0, \quad a_{22} < 0, \quad a_{12} a_{21} < 0.$$

Hence the zero solution to (19) is absolutely stable, because these inequalities are complied with.

SUMMARY

The results here show that normal analysis methods for linear systems can be used with many nonlinear systems, even if there are several nonlinearities. Popov [9] has observed that these results should be of value in harmonic linearization applications.

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*Similar systems are dealt with in [6], [7] and [8].

AN INTEGRAL ESTIMATE FOR SELECTING THE OPTIMUM PARAMETERS OF AN AUTOMATIC CONTROL SYSTEM WITH A GIVEN OVERSHOOT

A.I. Tupitsyn

(Khar'kov)

A combined integral estimate is used to find the optimum parameters of an automatic control system with a given overshoot. Formulas are given which define the response time approximately. Examples are given.

If the integral-square estimate $J_1 = \int_0^\infty [\Delta(t)]^2 dt$, where $\Delta(t)$ is deviation of the controlled quantity from a new steady value, is used to choose the optimum parameters of an automatic control system with a given overshoot, it is found [1-3] that the transient response is oscillatory and shows a large overshoot. That this is bound to be so has been demonstrated [4]. Large overshoots are sometimes impermissible or undesirable, and so other, more complex integral estimates are sought to give systems with better damping. A combined estimate (test) of this type is [1, 2, 5]

$$J_2 = J_1 + T^2 J_{01} = \int_0^\infty [\Delta(t)]^2 dt + T^2 \int_0^\infty [\Delta'(t)]^2 dt.$$

The extremum in this test is an exponential function of time-constant T, so we may expect to get a highly damped system if the parameters are chosen in accordance with this test. It is, in practice, difficult to use J_2, however, because we have to determine T in every separate case.

Here we deal with a combined test, $J_{11} = J_1 + \alpha/\Delta_0^4 J_1^2 J_{01}$, which gives optimum parameters for a given overshoot. Here α is a dimensionless parameter that defines the extremal transient, in particular, the overshoot.

We can find how the overshoot depends on α by considering the turning points in J_{11}.

The argument that gives us J_{11} is as follows: J_2 has as extremum the function $\Delta(t) = \Delta_0 e^{-t/T}$, in which T can be found from J_1:

$$J_1 = \int_0^\infty (\Delta_0 e^{-\frac{t}{T}})^2 dt = \frac{\Delta_0^2 T}{2},$$

whence

$$T = 2J_1/\Delta_0^2$$

We substitute for T in J_2 and get

$$J_{11} = J_1 + \frac{4}{\Delta_0^4} J_1^2 J_{01}.$$

It will be seen later on that this is, at its turning point, a function which is nearly exponential in form. The test can be generalized, however, if we replace the 4 by the dimensionless variable α.

We then have

$$J_{11} = J_1 + \frac{\alpha}{\Delta_0^4} J_1^2 J_{01}. \tag{1}$$

If now we are able to show how the transient response is related to α by considering the extrema in J_{11}, we may say that J_{11} can serve to choose the optimum parameters for a given overshoot.

The parameters given by J_{11} are optimal in the sense that the transient is briefest, because J_{11} is proportional to the time the transient lasts, for given overshoot and Δ_0; hence, minimum J_{11} means minimum response time.

This method, with J_{11}, is not the only one possible. Other tests can be derived by the method of dimensions, combined with giving dimensionless forms to the parameters that define the shapes of the extrema.

J_1 has dimensions $[\Delta]^2[t]$, and J_{01} has $[\Delta]^2[t]^{-1}$.

Consider $J_1 + \alpha J_1^y \Delta_0^z J_{01}$; we determine y and z from dimensional principles:

$$([\Delta]^2[t])^y [\Delta]^z [\Delta]^2 [t]^{-1} = [\Delta]^2[t].$$

Hence, $2y + z + 2 = 2$, $y - 1 = 1$, so $y = 2$, $z = 4$, which gives J_{11} above.

The Theoretical Extrema in $J_{11} = J_1 + (\alpha/\Delta_0^4)J_1^2 J_{01}$

The theoretical extrema in

$$J_{11} = J_1 + \frac{\alpha}{\Delta_0^4} J_1^2 J_{01}$$

are given (see Appendix) by (17) and (20), in which $\beta = f(\alpha)$ and $\beta_1 = f(\alpha)$ are given by (19) and (21).

Figure 1 shows $\beta = f(\alpha)$. Here α varies from 0 to ∞ as β goes from 0 to 2.176. The minimum in J_{11} occurs in this range.

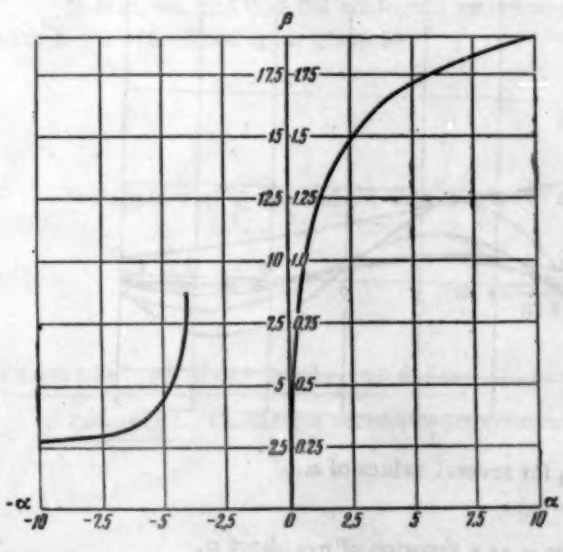

Fig. 1. The curve $\beta = f(\alpha)$.

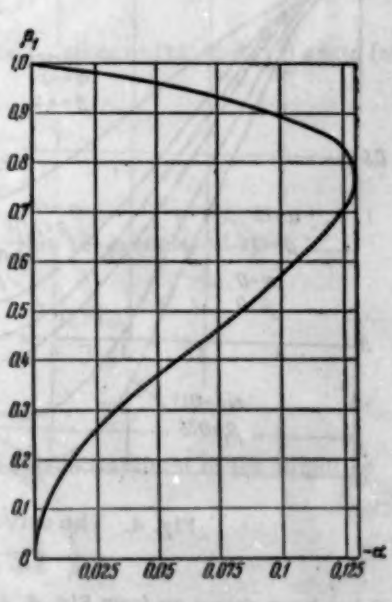

Fig. 2. The curve $\beta_1 = f(\alpha)$.

Now α varies from $-\infty$ to -4 as β varies from 2.176 to ∞, and we get the absolute minimum in J_{11}. Figure 2 shows $\beta_1 = f(\alpha)$. The relation is double-valued. The minimum in J_{11} corresponds to β_1 from 0 to 0.76, whereas values between 0.76 and 1 give a relative maximum.

Figure 3 shows

$$F_1(\nu) = \frac{\beta}{\operatorname{arc\,tg}\beta}\frac{1}{1+\beta^2\nu^2} \text{ и } F_2(\nu) = \frac{\beta_1}{\operatorname{Ar\,th}\beta_1}\frac{1}{1-\beta_1^2\nu^2}$$

for several values of β and β_1.

Trapezoidal frequency characteristics [6] give us $\Delta(\tau)/\Delta_0$, which is shown in Fig. 4 ($F_1(\nu)$ for $\beta = 4.4$ is not shown in Fig. 3).

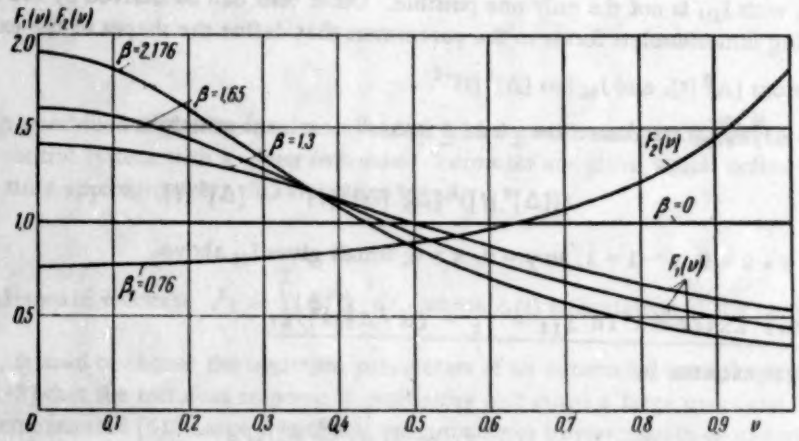

Fig. 3. The curves $F_1(\nu)$ and $F_2(\nu)$ for several values of β and β_1.

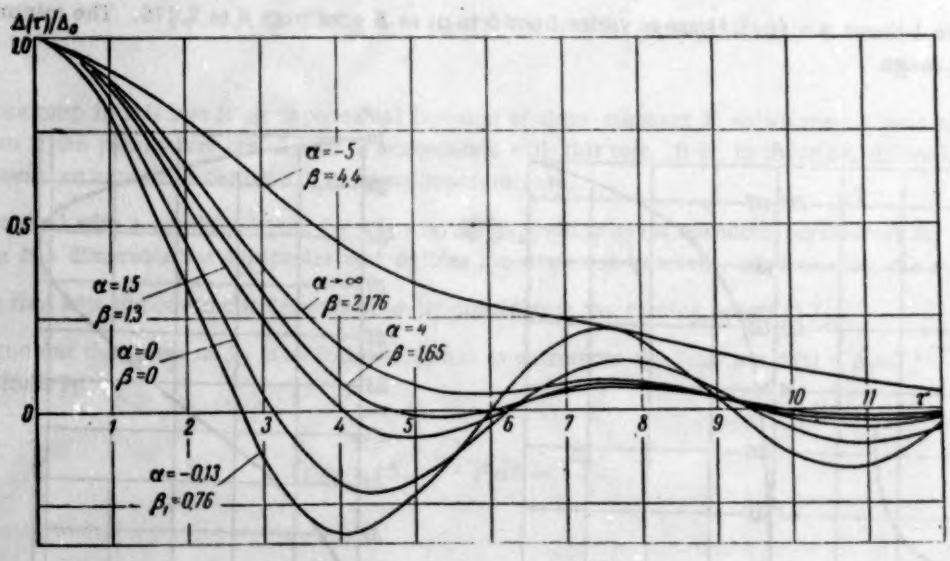

Fig. 4. The curve $\Delta(\tau)/\Delta_0$ for several values of α.

Figure 5 has been drawn up from Fig. 4, and gives us α as a function of overshoot σ.

The response time can be found roughly when the parameters corresponding to the minimum in J_{11} have been found.

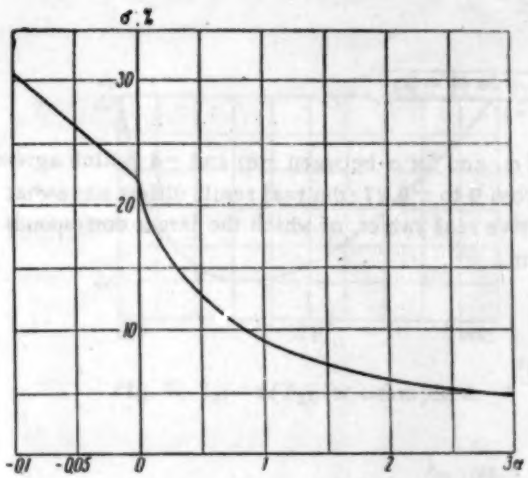

Fig. 5. Overshoot as a function of α.

The time τ_r from Fig. 4 is used to find ω_c: $t_r = \tau_r/\omega_c$.

Now ω_c is given by

$$\omega_c = \beta\Omega_c = j\beta_1\Omega_c, \tag{2}$$

where Ω_c is found from (13) (see Appendix):

$$\omega_c = \frac{\pi\Delta_0^2\beta A}{4\bar{J}_1 \operatorname{arc\,tg}^2\beta} = \frac{\pi\Delta_0^2\beta_1 A_1}{4\bar{J}_1 \operatorname{Ar\,th}^2\beta_1}, \tag{3}$$

$$\omega_c = \frac{4\bar{J}_{01}\beta \operatorname{arc\,tg}^2\beta}{\Delta_0^2\pi B} = \frac{4\bar{J}_{01}\beta_1 \operatorname{Ar\,th}^2\beta_1}{\Delta_0^2\pi B_1}, \tag{4}$$

$$\omega_c = \frac{\Delta_0\pi\beta}{2\bar{F}_c(0) \operatorname{arc\,tg}\beta} = \frac{\Delta_0\pi\beta_1}{2\bar{F}_c(0) \operatorname{Ar\,th}\beta_1}, \tag{5}$$

$$\omega_c = \frac{\Delta_0^2\pi\beta^2 A}{2\bar{J}_{11}(1+\beta^2)\operatorname{arc\,tg}^2\beta\,(A-2B)} = \frac{\Delta_0^2\pi\beta_1^2 A_1}{2\bar{J}_{11}(1-\beta_1^2)\operatorname{Ar\,th}^2\beta_1\,(A_1-2B_1)}. \tag{6}$$

Here $\bar{J}_{11}$ denotes the minimum J_{11}, and $\bar{J}_1$, $\bar{J}_{01}$ and $\bar{F}_c(0)$ denote the values of J_1, J_{01} and $F_c(0)$ that correspond to $\bar{J}_{11}$.

Here (3)-(5) derive from (18) and (16) (see Appendix). We have derived (6) by substituting for $\bar{J}$, $\bar{J}_{01}$ and α [from (18) and (19) in the Appendix] in (1).

All these expressions ought, in theory, to give the same ω_c. The real extrema differ from the theoretical ones, however, and so (2)-(6) will give differing values. The mean ω_c should be used.

The Estimate $J_{12} = J_1^2 J_{01}$

The relative importance of J_1 in J_{11} decreases as α becomes large. We may expect that the transient response corresponding to $\beta = 2.176$ will be given by $J_{12} = J_1^2 J_{01}$.

In fact, we find that the analogous variation problem for J_{12} gives us (16) and (17) again (see Appendix), with $\beta = 2.176$. Then Ω_c is given by

$$\Omega_c^2 = \frac{2\bar{J}_{01}}{\bar{J}_1}. \tag{7}$$

We again use (2)-(4) and (5) with $\beta = 2.176$ to get ω_c. Now (6) is replaced by

$$\omega_c = \frac{\Delta_0^6\pi^3\beta A^2 B}{64 \operatorname{arc\,tg}^6\beta\,\bar{J}_{12}} = \frac{0.84\,\Delta_0^6}{\bar{J}_{12}}. \tag{8}$$

Examples of Uses of J_{11} and J_{12}

Example 1. Consider a second-order system. Let the Laplace transform of the output be

$$\bar{x}(p) = \frac{1}{p^3 + 2kp + 1}\,\frac{1}{p}.$$

Then J_{11} is

$$J_{11} = \frac{4k^2+1}{4k} + \frac{\alpha}{64}\,\frac{(4k^2+1)^2}{k^3}.$$

We calculate dJ_{11}/dk and equate it to zero, and hence get

$$k^2_{opt} = \frac{2 + \alpha \pm \sqrt{(2+\alpha)^2 + 3\alpha(4+\alpha)}}{4(4+\alpha)}.$$

Hence k^2_{opt} is real and positive for all positive values of α, and for α between $-\infty$ and -4 (which agrees exactly with theory). The small negative values of α stretch from 0 to -0.27 (the real result differs somewhat from the theoretical). In this range k^2_{opt} actually has two positive real values, of which the larger corresponds to the minimum in J_{11}, and the smaller to the relative maximum.

We get J_{12} from

$$J_{12} = \frac{(4k^2+1)^2}{64k^2}.$$

The minimum occurs with $k_{opt} = 0.867$.

This value of k_{opt} follows from the general formula for the limit when $\alpha \to \infty$.

Figure 6 shows extremal curves for $\Delta(t) = 1 - x(t)$ for different values of α.

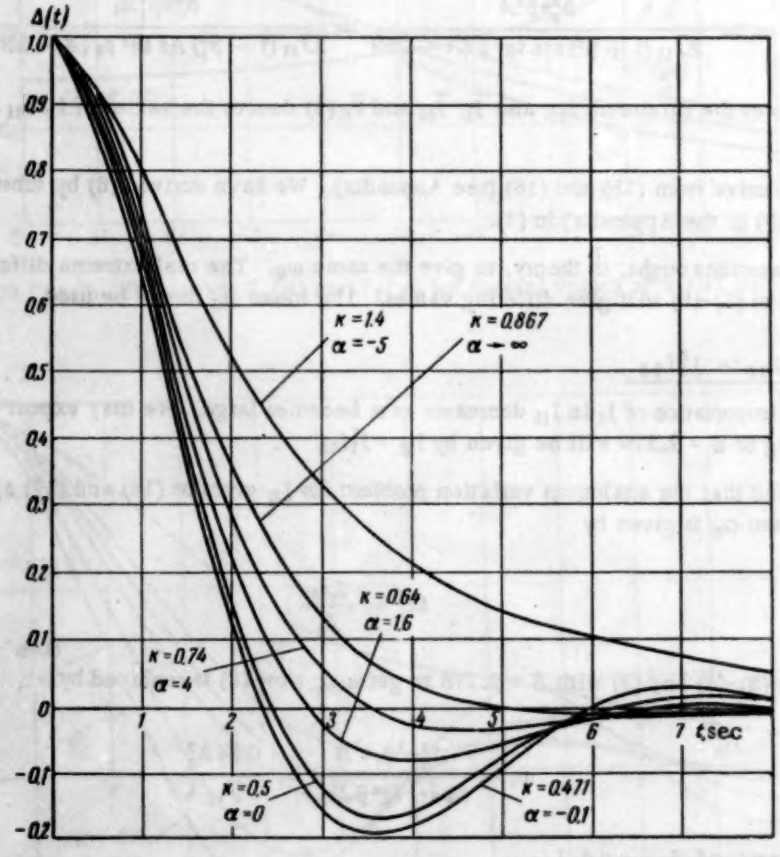

Fig. 6. $\Delta(t)$ for example 1.

We now compare the actual and theoretical response times for $k_{opt} = 0.867$.

We find ω_c from (2)-(5) and (8) as 1.54, 1.73, 1.37 and 2.16, respectively. The mean $\omega_c = 1.71$.

Then, from Fig. 4, we find $\tau_{ss\,10\%}$, the time at which $\Delta(\tau)$ deviates by not more than 10% from Δ_0: $\tau_{ss\,10\%} = 4.64$. Hence, the theoretical time is

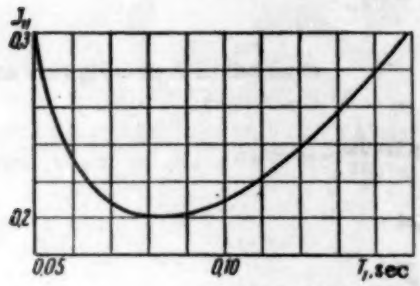

Fig. 7. $J_{11} = f(T_1)$ for example 2.

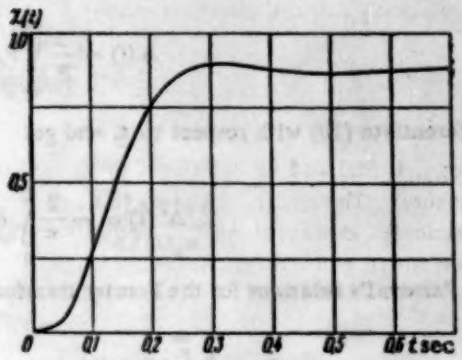

Fig. 8. x(t) for example 2.

$$t_{ss}\,10\% = 4.64/1.71 = 2.7 \text{ sec.}$$

The actual time given by Fig. 6 is 3.25 sec.

Example 2. We consider the optimal parameters for the stabilizing transformer in a self-adjusting motor-speed control. The circuit, the transfer function, the numerical values of the main parameters, and the expression for J_1, are given in [4].

For J_{01} we have

$$J_{01} = \frac{K^2}{2a_0}\,\frac{a_2a_3 - a_1a_4 + b_1^2a_0a_3}{a_1a_2a_3 - a_0a_3^2 - a_1^2a_4}.$$

We take $\alpha = 4$ in J_{11} in order to get only a small overshoot.

Figure 7 shows $J_{11} = f(T_1)$, which implies that J_{11} is a minimum when $T_1 = 0.08$ sec. Figure 8 shows the corresponding $x(t) = x_{ss} - \Delta(t)$ curve. The control time, $t_r\,10\%$, is 0.21 sec.

We now calculate the theoretical time. We have $\overline{J}_1 = 0.073$, $\overline{J}_{01} = 3.76$, $\overline{J}_{11} = 0.2$. The values of ω_c are: 19.0; 19.8; 17.8; 18.9; 21.6. The mean is $\omega_c = 19.4$.

The theoretical time is $t_r = 3.88/19.4 = 0.20$ sec.

SUMMARY

1. The integral estimate $J_{11} = J_1 + (\alpha/\Delta_0^4)J_1^2J_{01}$ is examined, and it is shown that this estimate can give optimum parameters corresponding to a preset overshoot.

2. J_{11} and J_{12} are used to find the optimum parameters for linear stationary systems; the actual results for the overshoot and response time agree well with those found from the theory.

The estimates can also be used in statistical dynamics problems.

APPENDIX

The Theoretical Extrema of $J_{11} = J_1 + (\alpha/\Delta_0^4)J_1^2J_{01}$

We find the theoretical extrema of J_{11} for various values of α by the method given in [4].

Let $\Delta(t)$ have a cosine Fourier transform [7]

$$F_c(\omega) = \int_0^\infty \Delta(t)\cos\omega t\,dt, \tag{9}$$

then $\Delta(t)$ is given by

$$\Delta(t) = \frac{2}{\pi}\int_0^\infty F_c(\omega)\cos t\omega\, d\omega. \tag{10}$$

We differentiate (10) with respect to t, and get

$$\Delta'(t) = -\frac{2}{\pi}\int_0^\infty F_c(\omega)\,\omega\sin t\omega\, d\omega.$$

We use Parseval's relations for the Fourier transform, and get

$$J_1 = \frac{2}{\pi}\int_0^\infty [F_c(\omega)]^2\, d\omega, \qquad J_{01} = \frac{2}{\pi}\int_0^\infty [F_c(\omega)]^2\,\omega^2 d\omega, \tag{11}$$

We pass to the final limits of integration with respect to ω, and use the approximate expressions for the integrals given by the formula for the mean rectangles, and get J_{11} as

$$J_{11} = \frac{2}{\pi}\frac{\omega_c}{n}\sum F_{ck}^2 + \frac{\alpha}{\Delta_0^4}\frac{4}{\pi^2}\left(\frac{\omega_c}{n}\right)^2\left(\sum F_{ck}^2\right)^2\frac{2}{\pi}\frac{\omega_c}{n}\sum\omega_k^2 F_{ck}^2.$$

The extra condition

$$\Delta_0 = \frac{2}{\pi}\int_0^{\omega_c} F_c(\omega)\, d\omega = \frac{2}{\pi}\frac{\omega_c}{n}\sum F_{ck} \tag{12}$$

implied by (10), used with Lagrange multipliers, gives us the function

$$F' = \frac{2}{\pi}\frac{\omega_c}{n}\sum F_{ck}^2 + \frac{\alpha}{\Delta_0^4}\frac{4}{\pi^2}\left(\frac{\omega_c}{n}\right)^2\left(\sum F_{ck}^2\right)^2\frac{2}{\pi}\frac{\omega_c}{n}\sum\omega_k^2 F_{ck}^2 + \lambda\frac{2}{\pi}\frac{\omega_c}{n}\sum F_{ck},$$

where λ is a Lagrange multiplier.

We find dF'/dF_{cj} and equate it to zero to get

$$F_{ck} + \frac{\alpha}{\Delta_0^4}2\bar{J}_1\bar{J}_{01}F_{ck} + \frac{\alpha}{\Delta_0^4}\bar{J}_1^2\omega_k^2 F_{ck} + \frac{\lambda}{2} = 0.$$

The continuous function $F_C(\omega)$ then is

$$F_c(\omega) = -\frac{{}^1/_2\,\lambda}{1 + \frac{\alpha}{\Delta_0^4}2\bar{J}_1\bar{J}_{01} + \frac{\alpha}{\Delta_0^4}\bar{J}_1^2\omega^2},$$

where $\bar{J}_1$ and $\bar{J}_{01}$ denote the values of J_1 and J_{01} that correspond to the extremum in J_{11}.

We use the notation

$$\Omega_c^2 = \frac{\Delta_0^4 + 2\alpha\bar{J}_1\bar{J}_{01}}{\alpha\bar{J}_1^2}, \tag{13}$$

$$\beta = \frac{\omega_c}{\Omega_c} \tag{14}$$

and the new dimensionless variable

$$\tau = \omega_c t, \quad \nu = \frac{\omega}{\omega_c}, \tag{15}$$

to get the expression

$$F_c(\nu) = -\frac{\lambda\Delta_0^4}{2\alpha\bar{J}_1^2\Omega_c^2}\,\frac{1}{1+\beta^2\nu^2}.$$

Then (12) gives us λ in the form

$$\Delta_0 = -\frac{\lambda\Delta_0^4\omega_c}{\alpha\bar{J}_1^2\Omega_c^2\pi}\int_0^1\frac{d\nu}{1+\beta^2\nu^2} = -\frac{\lambda\Delta_0^4\operatorname{arc\,tg}\beta}{\alpha\bar{J}_1^2\Omega_c\pi};$$

hence

$$\lambda = -\frac{\alpha\bar{J}_1^2\Omega_c\pi}{\Delta_0^3\operatorname{arc\,tg}\beta}.$$

Thus $F_c(\nu)$ is

$$F_c(\nu) = \frac{\Delta_0\pi\beta}{2\omega_c\operatorname{arc\,tg}\beta}\,\frac{1}{1+\beta^2\nu^2}. \tag{16}$$

Then (10) gives us $\Delta(\tau)$ as

$$\Delta(\tau) = \frac{\Delta_0\beta}{\operatorname{arc\,tg}\beta}\int_0^1\frac{\cos\tau\nu\,d\nu}{1+\beta^2\nu^2}. \tag{17}$$

Hence (17) gives the extremum in J_{11} as a function of the dimensionless variable $\tau = \omega_c t$.

We relate β to α by finding expressions for $\bar{J}_1$ and $\bar{J}_{01}$ from (11) and (16)

$$\bar{J}_1 = \frac{2}{\pi}\int_0^{\omega_c}[F_c(\omega)]^2\,d\omega = \frac{\Delta_0^2\beta\pi A}{4\omega_c\operatorname{arc\,tg}^2\beta},\quad \bar{J}_{01} = \frac{2}{\pi}\int_0^{\omega_c}[F_c(\omega)]^2\,\omega^2\,d\omega = \frac{\Delta_0^2\pi\omega_c B}{4\beta\operatorname{arc\,tg}^2\beta}, \tag{18}$$

where

$$A = \operatorname{arc\,tg}\beta + \frac{\beta}{1+\beta^2},\quad B = \operatorname{arc\,tg}\beta - \frac{\beta}{1+\beta^2}.$$

Substituting in (13), we have

$$\Omega_c^2 = \frac{\Delta_0^4 + 2\alpha\,\dfrac{\Delta_0^4\pi^2 AB}{16\operatorname{arc\,tg}^4\beta}}{\dfrac{\alpha\Delta_0^4\beta_2^2\pi^2A^2}{16\omega_c\operatorname{arc\,tg}^4\beta}},$$

whence, from (14), we have

$$\alpha = \frac{16\operatorname{arc\,tg}^4\beta}{\pi^2A(A-2B)}. \tag{19}$$

This variational problem also has solutions if β is imaginary.

We put $\beta = j\beta_1$ in (17) and (19), and get

$$\Delta(\tau) = \frac{\Delta_0\beta_1}{\operatorname{Ar\,th}\beta_1}\int_0^1\frac{\cos\tau\nu d\nu}{1-\beta_1^2\nu^2}, \tag{20}$$

$$\alpha = -\frac{16\operatorname{Ar\,th}^4\beta_1}{\pi^2A_1(A_1-2B_1)}, \tag{21}$$

where $A_1 = \operatorname{Ar\,th}\beta_1 + \beta_1/(1-\beta_1^2)$, $B_1 = \operatorname{Ar\,th}\beta_1 - \beta_1/(1-\beta_1^2)$, and β_1 is a real quantity.

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USE OF AN INTEGRAL-SQUARE ESTIMATE TO DETERMINE THE OPTIMAL PARAMETERS OF AN AUTOPILOT WITH RATE FEEDBACK*

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(Moscow)

The stabilization of an aircraft by an autopilot with rate feedback is studied. Analytic formulas are deduced which define the optimum values of the transfer numbers for the autopilot in terms of the aircraft's aerodynamic parameters and of the flight conditions. Lag in the controls is allowed for. The integral-square estimate is minimized to yield the optimum stabilization parameters.

To define the optimum parameters for the autopilot is one of the most common tasks in designing aircraft controls. Aircraft have large speed ranges, and their above-ground heights vary greatly; the calculations needed can therefore be lengthy and troublesome.

An analytic method of defining the parameters (i.e., the transfer numbers) is thus of some interest.

The problem is as follows: the aircraft's aerodynamic and geometrical parameters are known (e.g., for stabilization in a longitudinal plane), and so are the flight conditions; and from these data the optimum transfer numbers must be found. The work involved should be minimized.

This analytic problem is solved here for an autopilot with rate feedback. The idealized law is of the form

$$T\Delta\ddot{\delta} + \Delta\dot{\delta} = k_0\Delta\vartheta + k_1\Delta\dot{\vartheta} + k_2\Delta\ddot{\vartheta},$$

where $\Delta\vartheta = \vartheta - \vartheta_0$ is the deviation of the course angle from the set value, $\Delta\delta$ is a variation in the rudder setting, T specifies the lag in the control, and k_0, k_1 and k_2 are the transfer numbers.

The motion about the center of gravity, when the general motion is longitudinal, is described by

$$\Delta\ddot{\vartheta} + \frac{1}{T_\omega}\Delta\dot{\vartheta} + \frac{1}{T_{\dot\alpha}}\Delta\dot{\alpha} + k_\alpha\Delta\alpha = -k_\delta\Delta\delta. \tag{1}$$

Here $\Delta\alpha$ is a variation in the angle of attack,

$$\frac{1}{T_\omega} = \frac{m_z^{\omega z}\frac{\rho V}{2}Sb_A^2}{J_z}, \tag{2}$$

*Read at the seminar on automatic control at the Institute of Automation and Remote Control in 1958.

$$\frac{1}{T_\alpha} = \frac{m_z^{\dot\alpha}\frac{\rho V}{2} S b_A^2}{J_z}, \tag{3}$$

$$k_\alpha = \frac{m_z^{\alpha}\frac{\rho V^2}{2} S b_A}{J_z}, \tag{4}$$

$$k_\delta = \frac{m_z^{\delta}\frac{\rho V^2}{2} S b_A}{J_z}. \tag{5}$$

In (2)-(5) ρ is the density of the air, V is the speed, S is the wing area, b_A is the mean aerodynamic chord of a wing, J_Z is the aircraft's moment of inertia, $m_Z^{\omega_Z}$ is a damping coefficient, $m_Z^{\dot\alpha}$ is the aerodynamic damping coefficient, m_Z^{α} is the static stability coefficient, and m_Z^{δ} is the aileron coefficient.

We assume that the direction of the general motion does not change while the transient about the center of gravity is dying away, and so we can assume that

$$\Delta\vartheta \approx \Delta\alpha \quad \text{and} \quad \Delta\dot\vartheta \approx \Delta\dot\alpha.$$

We then have

where $$\Delta\ddot\vartheta + \frac{1}{T_\Omega}\Delta\vartheta + k_\alpha\Delta\vartheta = -k_\delta\Delta\delta, \quad \frac{1}{T_\Omega} = \frac{1}{T_\omega} + \frac{1}{T_\alpha}.$$

A fourth-order equation describes the closed-loop system formed by aircraft and autopilot:

$$T\Delta\vartheta^{IV} + \left(1 + \frac{T}{T_\Omega}\right)\Delta\vartheta''' + \left(\frac{1}{T_\Omega} + k_2k_\delta + Tk_\alpha\right)\Delta\vartheta'' + (k_\alpha + k_1k_\delta)\Delta\vartheta' + k_0k_\delta\Delta\vartheta = 0. \tag{6}$$

Gurvits' determinants for the system are

$$\Delta_0 = \left(1 + \frac{T}{T_\Omega}\right)\left(\frac{1}{T_\Omega} + k_2k_\delta + Tk_\alpha\right)(k_\alpha + k_1k_\delta) - T(k_\alpha + k_1k_\delta)^2 - k_0k_\delta\left(1 + \frac{T}{T_\Omega}\right)^2, \tag{7}$$

$$\Delta_1 = \left(1 + \frac{T}{T_\Omega}\right)\left(\frac{1}{T_\Omega} + k_2k_\delta + Tk_\alpha\right) - T(k_\alpha + k_1k_\delta). \tag{8}$$

We take the minimum in the integral of the deviation squared (the integral-square deviation) as criterion for the optimal choice of transfer numbers (the integral test).

This indirect method may give transfer numbers that imply excessive oscillation, but which are suitable for subsequent electronic analog studies [1].

The integral-square test is $J_0 = \int_0^\infty \Delta\vartheta^2 dt$, and any of the usual methods [1-3] can be used to derive it.

We assume that we have at t = 0 a typical initial perturbation $\Delta\vartheta = \Delta\vartheta_0$, $\Delta\dot\vartheta = \Delta\ddot\vartheta = 0$, and that as $t \to \infty$ the stabilization process goes to completion; the integral test then takes the form

$$\bar{J}_0 = \frac{J_0}{\frac{\Delta\vartheta_0^2}{2}} = \frac{k_\alpha}{k_2 k_\delta} + \frac{k_1}{k_0} + \frac{\left(\frac{1}{T_\Omega} + k_2 k_\delta + T k_\alpha\right)^2 \left(1 + \frac{T}{T_\Omega}\right) - T\,(k_\alpha + k_1 k_\delta)\left(\frac{1}{T_\Omega} + k k_\delta + T k_\alpha\right) - T k_0 k_\delta \left(1 + \frac{T}{T_\Omega}\right)}{\left(\frac{1}{T_\Omega} + k_2 k_\delta + T k_\alpha\right)(k_\alpha + k_1 k_\delta)\left(1 + \frac{T}{T_\Omega}\right) - T\,(k_0 + k_1 k_\delta)^2 - k_0 k_\delta \left(1 + \frac{T}{T_\Omega}\right)^2}. \quad (9)$$

Then (7) and (8) give

$$\bar{J}_0 = \frac{k_\alpha}{k_0 k_\delta} + \frac{k_1}{k_0} + \frac{\left(\frac{1}{T_\Omega} + k_2 k_\delta + T k_\alpha\right)\Delta_1 - T k_0 k_\delta \left(1 + \frac{T}{T_\Omega}\right)}{\Delta_0}. \quad (10)$$

Hence, (10) gives us the integral test in terms of the autopilot's transfer numbers, and of the aircraft's aerodynamic parameters.

This rather cumbersome form can be simplified by applying the first condition for the test to be a minimum:

$$\frac{\partial J_0}{\partial k_2} = 0.$$

This condition is

$$\Delta_1 \left[\Delta_0 - k_0 k_\delta \left(1 + \frac{T}{T_\Omega}\right)^2\right] = 0. \quad (11)$$

Now, if Gurvits' Δ_1 is zero, $\bar{J}_0$ has a maximum (the system is not stable; the integral diverges).

The test will have a minimum if

$$\Delta_0 = k_0 k_\delta \left(1 + \frac{T}{T_\Omega}\right)^2. \quad (12)$$

Then (12) gives

$$k_2 = \frac{2k_0\left(1 + \frac{T}{T_\Omega}\right)}{k_\alpha + k_1 k_\delta} - \frac{T k_\alpha}{k_\delta} - \frac{1}{T_\Omega k_\delta} + \frac{T\,(k_\alpha + k_1 k_\delta)}{k_\delta \left(1 + \frac{T}{T_\Omega}\right)}. \quad (13)$$

We use (12) to simplify the form of the test:

$$\bar{J}_0 = \frac{k_\alpha}{k_0 k_\delta} + \frac{k_1}{k_0} + \frac{4k_0 k_\delta \left(1 + \frac{T}{T_\Omega}\right)}{(k_\alpha + k_1 k_\delta)^2} + \frac{T}{1 + \frac{T}{T_\Omega}}. \quad (14)$$

From (14) we readily get the second condition for the test to be a minimum $\partial J_1/\partial k_1 = 0$:

$$k_\alpha + k_1 k_\delta = 2\,(k_0 k_\delta)^{1/2}\left(1 + \frac{T}{T_\Omega}\right)^{1/2}$$

or

$$k_1 = 2k_0 \left(\frac{1 + \frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{1/2} - \frac{k_\alpha}{k_\delta}. \quad (15)$$

We use (15) to get k_2:

$$k_2 = k_0\left(\frac{1+\frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{3/4} - \frac{Tk_\alpha}{k_\delta} - \frac{1}{T_\Omega k_\delta} + \frac{2Tk_\delta^{3/4}}{\left[k_\delta\left(1+\frac{T}{T_\Omega}\right)^2\right]^{1/4}}. \tag{16}$$

Then, with k_1 and k_2 given by (15) and (16), the expression for $\overline{J}_\vartheta$ becomes

$$\overline{J}_0 = 3\left(\frac{1+\frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{1/4} + \frac{T}{1+\frac{T}{T_\Omega}}. \tag{17}$$

Thus k_1 and k_2 can be found in terms of k_0 and of the aircraft's aerodynamic parameters. It remains to find an analytic condition to guide us to a reasonable choice of k_0.

Now (17) shows that the test follows a cubic hyperbola law (Fig. 1) with respect to k_0.

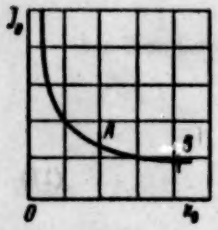

Fig. 1.

The test varies very slowly with k_0 in region AB, and so there is no point in increasing k_0 further. Now the drive power must be increased in order to increase k_0 (the rate coefficient for the rudder drive), while the transient response is not much affected.

The $J_\vartheta = f(k_0)$ curve has a characteristic point of maximum curvature. The best k_0 is somewhat greater than the k_0 that corresponds to this point of maximum curvature on $\overline{J}_\vartheta = f(k_0)$.

The curvature is given by

$$\frac{1}{R} = \frac{|\overline{J}_0''|}{\left(1+\overline{J}_0'^2\right)^{3/4}}.$$

The maximum curvature occurs at a k_0 given by

$$k_{0\max} = \left(\frac{5}{8}\right)^{3/4}\left(\frac{1+\frac{T}{T_\Omega}}{k_\delta}\right)^{1/4} \approx 0.9\left(\frac{1+\frac{T}{T_\Omega}}{k_\delta}\right)^{1/4}. \tag{18}$$

The criterion for k_0 is the inequality

$$k_0 \geqslant (2-4)\left(\frac{1+\frac{T}{T_\Omega}}{k_\delta}\right)^{1/4}. \tag{19}$$

Our analog studies have shown that such a choice of k_0 is quite satisfactory.

Hence, the optimum parameters of an astatic autopilot described by the equation

$$T\Delta\ddot{\delta} + \Delta\dot{\delta} = k_0\Delta\vartheta + k_1\Delta\dot{\vartheta} + k_2\ddot{\vartheta},$$

are given us by expressions

for k_0

$$k_0 \approx (2-4)\left(\frac{1+\frac{T}{T_\Omega}}{k_\delta}\right)^{1/4};$$

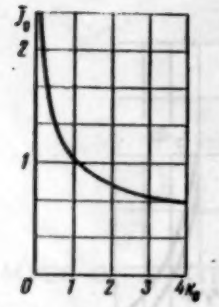

Fig. 2.

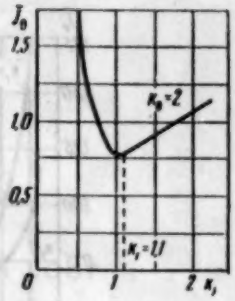

Fig. 3.

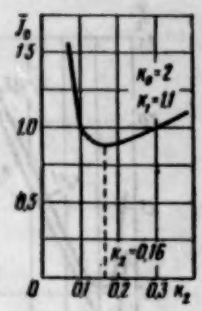

Fig. 4.

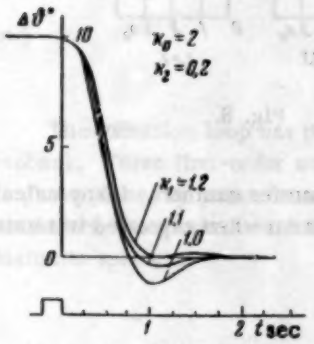

Fig. 5.

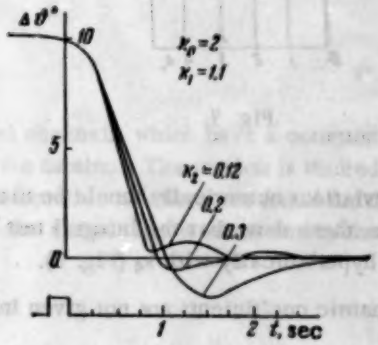

Fig. 6.

for k_1

$$k_1 = 2k_0\left(\frac{1+\frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{1/2} - \frac{k_\alpha}{k_\delta};$$

and for k_2

$$k_2 = k_0\left(\frac{1+\frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{1/2} - \frac{Tk_\alpha}{k_\delta} - \frac{1}{T_\Omega k_\delta} + \frac{2Tk_0^{3/2}}{k_\delta^{1/2}\left(1+\frac{T}{T_\Omega}\right)^{3/2}}. \qquad (20)$$

The values are refined by using an electronic analog before design values are suggested.

Consider, for example, the optimum transfer numbers for stabilizing a hypothetical aircraft in banking We assume that $k_\alpha = 0$, $k_\delta = 30$ sec^{-2}, $1/T_\Omega = 2.5$ sec^{-1} and $T = 0.1$ sec.

Figure 2 shows how the integral test varies with k_0. Then $k_{0\max}$ is 0.5 sec^{-1}.

We make the transients tolerable by increasing $k_{0\max}$ fourfold, and get $k_0 = 2$ sec^{-1}. We then use (15) to get the optimum value for the derivative coefficient, $k_1 = 1.1$.

Figure 3 shows how the normalized integral test varies with k_1.

We use (16) to find k_2 (the transfer number for the second derivative of the control signal) as 0.16 sec.

Figure 4 shows how $\bar{J}_0$ varies with k_2.

Figures 5 and 6 show transient responses for $k_0 = 2$, with various combinations of values for k_1 and k_2.

It is clear that these formulas give transfer numbers that correspond to reasonable control response.

Some results given in [5] may serve as the second example illustrating our method. An autopilot described by $\delta = k_0\Delta\vartheta + k_1\Delta\dot{\vartheta} + k_2\Delta\ddot{\vartheta}$ was considered, and it was suggested that a reasonable method of constructing sets of

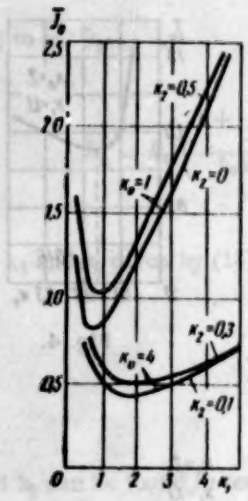

Fig. 7.

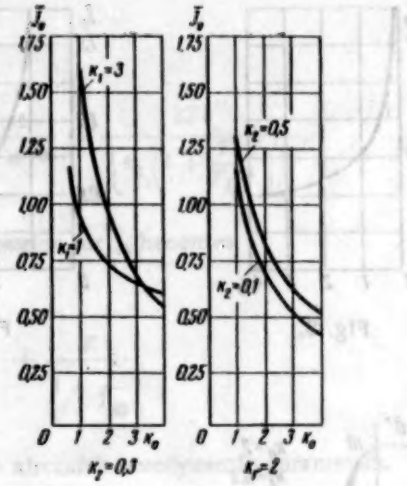

Fig. 8.

integral-square deviations numerically should be used to define the transfer numbers. Many calculations are given in this paper; these show that the integral test has a sharp minimum when expressed in terms of k_1 (Fig. 7), and that $\bar{J}_0$ varies hyperbolically with k_0 (Fig. 8).

The aerodynamic coefficients are not given in the paper.

SUMMARY

1. Approximate formulas are given for the transfer numbers of an autopilot with rate feedback; these formulas are in terms of the flight conditions and aerodynamic parameters:

$$k_0 \approx (2-4)\left(\frac{1+\frac{T}{T_\Omega}}{k_\delta}\right)^{1/3},$$

$$k_1 = 2k_0\left(\frac{1+\frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{1/3} - \frac{k_\alpha}{k_\delta},$$

$$k_2 = k_0\left(\frac{1+\frac{T}{T_\Omega}}{k_0 k_\delta}\right)^{2/3} - \frac{Tk_\alpha}{k_\delta} - \frac{1}{T_\Omega k_\delta} + \frac{2Tk_0^{2/3}}{k_\delta^{1/3}\left(1+\frac{T}{T_\Omega}\right)^{1/3}}.$$

2. The formulas can be used to give good approximations, which may be used in making a final choice for the numbers when the complete set of equations is examined on analogs.

The authors wish to thank A.M. Letov and B.V. Shirokorad for very valuable advice.

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THE DYNAMICS OF A VIBRATION LOOP IN AN ELECTRICAL SERVO DURING FREE OSCILLATIONS

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The vibration loop has two parallel control channels, which have a common feedback. Three first-order equations describe the motion. The motion is studied in the phase plane, by point-transform methods. Bifurcation relations are found for the parameters; these relations define the bounds to the various forms of motion in the parameter space.

Vibration servos are often used in automatic controls. The vibration loop is an important part of any such servo, because it effects vibrational linearization. Internal loops are often used for the purpose; these loops effect local feedback between several amplifying units. The feedbacks have lags.

The rate of change at the servo's output is usually very much less than the rates of change within the vibration loop. The system can thus be divided, in principle, into two independent parts; the vibration loop is considered as a closed dynamic system, and the servo as a whole is treated as though the vibration loop were a static sampled-data unit.

The stability of any such loop must be examined. Such a loop is essentially nonlinear, and so the stability problem is one in nonlinear theory. Andronov's [1, 2] point-transforms method, and phase-space methods, are used here to solve the problem. Several papers [3, 4] deal with phase-space methods used to solve nonlinear vibration-loop problems, but in all cases the loops had unbranched input circuits.

The Equations of Motion

We consider the vibration loop in an electrical autopilot [5]. Figure 1 shows the loop's electrical system. The input U is fed to the grid of tube 1, which amplifies it and feeds the double triode 2. The plates of triode 2 are ac-fed in opposite phases. The tubes conduct during the positive half-cycles only. The second half of the tube 2 conducts when the signal changes in sign. Relays P_1 and P_2, with capacitors C_1 and C_2, smooth the current pulses in the plate circuits. The relay contacts kP_1 and kP_2 operate the effector, and close the feedback circuit, whose signal is fed to the grid of tube 1, where it adds to the input. The circuit R_3, C_3 in the feedback loop causes a lag in the feedback signal when the relays operate and release.

The following assumptions are made in solving the mathematical problem for the system.

1. That the first stage is linear and lag-free.

2. That the second stage is lag-free, with static characteristics $F^*_1(U_1)$ and $F^*_2(U_1)$ for the two halves of tube 2. Figure 2 shows $F^*_1(U_1)$ and $F^*_2(U_1)$ (U_1 is the voltage applied to the grid of tube 2).

3. The plate circuits of tube 2 have inductance and resistance only, with different values for positive and negative signals, i.e., when tube 2 is conducting and not conducting.

4. The contacts' effects are described by the nonlinear functions $U_0\Phi(i_1)$ and $U_0\Phi(i_2)$ shown in Fig. 3 (i_1 and i_2 are the currents in the plate circuits of tube 2).

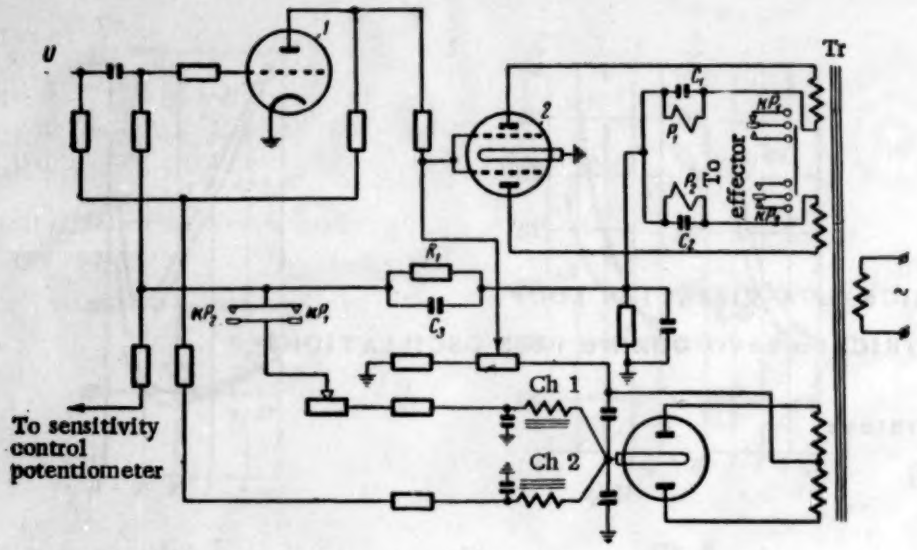

Fig. 1.

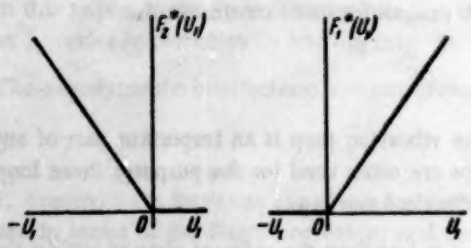

Fig. 2.

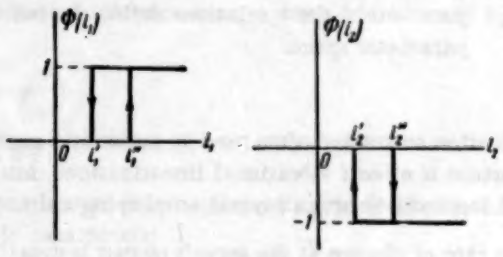

Fig. 3.

5. The resistor R_1 and capacitor C_3 are fixed.

The equations corresponding to this model (for U = 0) are

$$U_1 = k_1 \Delta U_1, \tag{1}$$

$$\left.\begin{aligned} L\frac{di_1}{dt} + Ri_1 &= k_2 U_1 \\ L'\frac{di_2}{dt} + R'i_2 &= 0 \end{aligned}\right\} \quad \text{for} \quad U_1 > 0, \tag{2}$$

$$\left.\begin{aligned} L'\frac{di_1}{dt} + R'i_1 &= 0 \\ L\frac{di_2}{dt} + Ri_2 &= -k_2 U_1 \end{aligned}\right\} \quad \text{for} \quad U_1 < 0, \tag{3}$$

$$C_3 R_1 \frac{dU_{fb}}{dt} + U_{fb} = U_0 \Phi(i_1, i_2), \tag{4}$$

$$\Delta U_1 = -bU_{fb} \tag{5}$$

Here ΔU_1 is the voltage on the grid of tube 1, L and R are the inductance and resistance in the plate circuit when tube 2 is working, L' and R' are the same when tube 2 is not conducting, k_1 and k_2 are the gains of the first and second stages; $k_1 = U_1 max/\Delta U_1 max$, $k_2 = F^*_1(U_1)max/U_1 max$; $\underline{b}$ is the feedback coefficient; U_{fb} is the feedback signal and U_0 is the voltage supplying the feedback circuit. Function $\Phi(i_1, i_2)$ can be put in the form

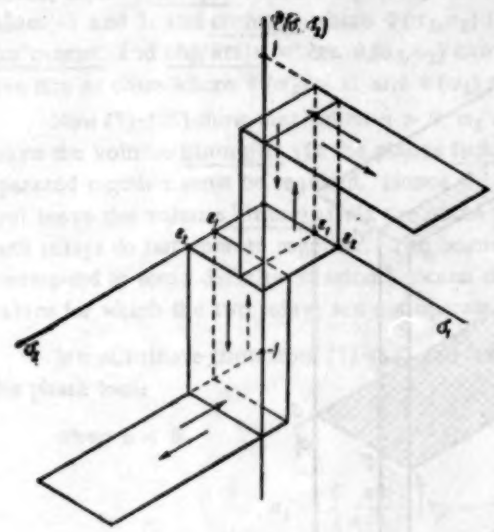

Fig. 4.

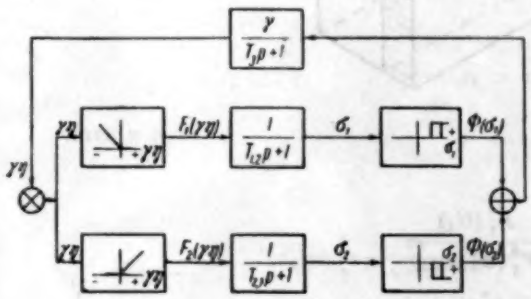

Fig. 5.

$$\Phi(i_1, i_2) = \begin{cases} 1 \text{ for } \Phi(i_1) = 1,\ \Phi(i_2) = 0, \\ 0 \text{ for } \Phi(i_1) = 0,\ \Phi(i_2) = 0, \\ -1 \text{ for } \Phi(i_1) = 0,\ \Phi(i_2) = -1. \end{cases} \quad (6)$$

Here $\Phi(i_1, i_2)$ has been put in this form on the assumption that P_1 and P_2 do not operate together.

We introduce relative quantities

$$\zeta = \frac{\Delta U_1}{\Delta U_{1\max}},\ \nu = \frac{U_1}{U_{1\max}},\ \sigma_1 = \frac{i_1}{i_{1\max}},$$

$$\sigma_2 = \frac{i_2}{i_{2\max}},\ \eta = \frac{U_{oc}}{U_0},$$

derive $\Phi(\sigma_1)$, $\Phi(\sigma_2)$, $\Phi(\sigma_1, \sigma_2)$ and eliminate ζ and ν; the system of equations then becomes

$$\left.\begin{aligned} T_1 \frac{d\sigma_1}{dt} + \sigma_1 &= -\gamma\eta \\ T_2 \frac{d\sigma_2}{dt} + \sigma_2 &= 0 \end{aligned}\right\} \text{ for } \eta < 0, \quad (7)$$

$$\left.\begin{aligned} T_2 \frac{d\sigma_1}{dt} + \sigma_1 &= 0 \\ T_1 \frac{d\sigma_2}{dt} + \sigma_2 &= \gamma\eta \end{aligned}\right\} \text{ for } \eta > 0, \quad (8)$$

$$T_3 \frac{d\eta}{dt} + \eta = \Phi(\sigma_1, \sigma_2), \quad (9)$$

$$\Phi(\sigma_1, \sigma_2) = \begin{cases} 1 \text{ for } \Phi(\sigma_1) = 1,\ \Phi(\sigma_2) = 0, \\ 0 \text{ for } \Phi(\sigma_1) = 0,\ \Phi(\sigma_2) = 0, \\ -1 \text{ for } \Phi(\sigma_1) = 0,\ \Phi(\sigma_2) = -1. \end{cases} \quad (10)$$

Figure 4 shows $\Phi(\sigma_1, \sigma_2)$:

$$\Phi(\sigma_1) = \begin{cases} 1 \text{ for } \sigma_1 > \varepsilon_1 & \Phi(\sigma_{10}) = 1, \\ 0 \text{ for } \sigma_1 < \varepsilon_2 & \Phi(\sigma_{10}) = 0, \end{cases} \quad (11)$$

$$\Phi(\sigma_2) = \begin{cases} -1 \text{ for } \sigma_2 > \varepsilon_1 & \Phi(\sigma_{20}) = -1, \\ 0 \text{ for } \sigma_2 < \varepsilon_2 & \Phi(\sigma_{20}) = 0. \end{cases} \quad (12)$$

Here σ_{10}, σ_{20} are initial values, and

$$\varepsilon_1 = \frac{i_1'}{i_{1\max}} = \frac{i_2'}{i_{2\max}},\ \varepsilon_2 = \frac{i_1''}{i_{1\max}} = \frac{i_2''}{i_{2\max}},\ T_1 = \frac{L}{R},\ T_2 = \frac{L'}{R'},\ T_3 = R_1 C_3,\ \gamma = b\,\frac{U_0}{\Delta U_{1\max}}.$$

This set of equations has been put down with due regard to the nonlinear functions $F_1(\gamma\eta)$ and $F_2(\gamma\eta)$, which have the forms

$$F_1(\gamma\eta) = \begin{cases} -\gamma\eta & \text{for } \eta < 0, \\ 0 & \text{for } \eta > 0, \end{cases} \quad (13)$$

$$F_2(\gamma\eta) = \begin{cases} \gamma\eta & \text{for } \eta > 0, \\ 0 & \text{for } \eta < 0. \end{cases} \quad (14)$$

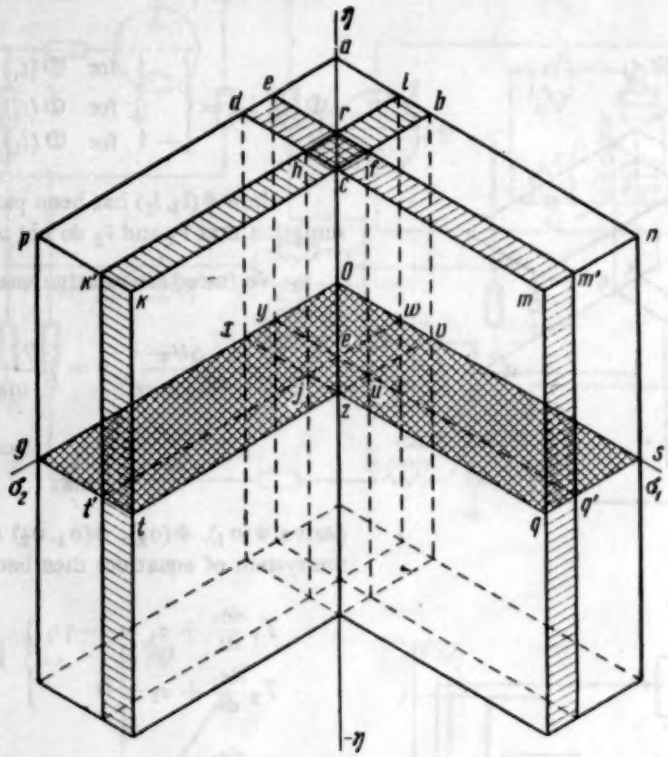

Fig. 6.

Here

$$F_1(\gamma\eta) = -F_1(\nu) = -\frac{F_1^*(U_1)}{F_1^*(U_1)_{\max}}$$

$$F_2(\gamma\eta) = -F_2(\nu) = -\frac{F_2^*(U_1)}{F_2^*(U_1)_{\max}},$$

$$F_1^*(U_1)_{\max} = i_{1\max}R,$$
$$F_2^*(U_1)_{\max} = i_{2\max}R.$$

Hence, the loop is specifiable in terms of the six parameters T_1, T_2, T_3, γ, ϵ_1, ϵ_2.

Figure 5 shows the structural diagram of a dynamic analog to the loop. There are three aperiodic links, and four nonlinear characteristics. Phase-space and point-transform methods are best used with a nonlinear system of this type. The units in the loop have piecewise-linear response, so the phase space splits up into several regions, in each of which we have a set of linear differential equations, whose right halves differ from one region to another. The equations for the boundaries are known, and so we can follow the phase loci throughout the space, by transforming one surface to another point-by-point. We can establish the structural bifurcation in the phase space exactly by point-transform methods, for different ratios between the parameters, and can thus find what types of oscillation can exist.

The system (7)-(12) shows that the phase space is three-dimensional (σ_1, σ_2, η). The equations for $\eta > 0$ and $\eta < 0$ differ, so the plane $\eta = 0$ cuts the space into two parts symmetrical about the origin. The σ_1, σ_2, η phase space is bounded by the planes $\sigma_1 = 0$, $\sigma_2 = 0$, $\eta = 1$, $\eta = -1$, $\sigma_1 = \gamma$, $\sigma_2 = \gamma$, which minimum and maximum values may be taken as being variable. The phase space is divided into several different regions by switching surfaces, because $\Phi(\sigma_1, \sigma_2)$ is nonlinear. These surfaces are planes, and so the space is cut up into several rectangular parallelepipeds, which overlap because $\Phi(\sigma_1, \sigma_2)$ has regions in which it is not single-valued. Figure 6 shows the phase space. The internal region is bounded by the planes $\sigma_1 = \epsilon_2$ and $\sigma_2 = \epsilon_2$, because from $\Phi(\sigma_1, \sigma_2)$ we have excluded the region in which $\Phi(\sigma_1)$ and $\Phi(\sigma_2)$ are, for all values of σ_1 and σ_2, simultaneously equal to +1 and −1.

Consider the region $\eta > 0$ in the σ_1, σ_2, η space. There are five regions where the functions are not single-valued, namely derhjxyl, where $\Phi(\sigma_1,\sigma_2)$ can take the values -1 and 0; fribvulw, where $\Phi(\sigma_1,\sigma_2)$ can take the values $+1$ and 0; and chrfuzjl, where $\Phi(\sigma_1,\sigma_2)$ may take the values $+1$, 0 and -1. There are also regions cfm'mqzuq' and chk'ktzjt' where $\Phi(\sigma_1,\sigma_2)$ can have the values $+1$ and -1, and so some initial deviations can give rise to cases where $\Phi(\sigma_1) = +1$ and $\Phi(\sigma_2) = -1$ simultaneously.

Now (7)-(12) show that, when $\eta > 0$, σ_2 and η increase, whereas σ_1 decreases; hence the phase loci can leave the volume hinmqjws via the planes iwjh and mqjh. In the latter case, the region where both relays are operated together must be reached. Hence we consider only the motion for initial deviations such that the phase loci leave the volume himnqjws via the plane iwjh. This region is the one to which applies the condition that both relays do not operate together. The bound to this region is the locus through hj. This limiting case will correspond to some definite relation between the loop's parameters. We may delineate a region of parameter values for which the two relays are not operated together, for a given set of initial deviations.

We eliminate time from (7)-(12) and integrate to derive the surfaces in phase space that intersect to give the phase loci:

when $\eta < 0$

$$\sigma_1 = \left\{\frac{a\gamma}{a-1}[\eta_0 - \Phi(\sigma)] + \sigma_{10} + \gamma\Phi(\sigma)\right\}\left[\frac{\eta - \Phi(\sigma)}{\eta_0 - \Phi(\sigma)}\right]^a - \tag{15}$$

$$- \frac{a\gamma}{a-1}[\eta - \Phi(\sigma)] - \gamma\Phi(\sigma),$$

$$\sigma_2 = \sigma_{20}\left[\frac{\eta - \Phi(\sigma)}{\eta_0 - \Phi(\sigma)}\right]^b; \tag{16}$$

when $\eta > 0$

$$\sigma_2 = \left\{-\frac{a\gamma}{a-1}[\eta_0 - \Phi(\sigma)] + \sigma_{20} - \gamma\Phi(\sigma)\right\}\left[\frac{\eta - \Phi(\sigma)}{\eta_0 - \Phi(\sigma)}\right]^a + \tag{17}$$

$$+ \frac{a\gamma}{a-1}[\eta - \Phi(\sigma)] + \gamma\Phi(\sigma),$$

$$\sigma_1 = \sigma_{10}\left[\frac{\eta - \Phi(\sigma)}{\eta_0 - \Phi(\sigma)}\right]^b, \tag{18}$$

where $a = T_3/T_1$, $b = T_3/T_1$, σ_{10}, σ_{20}, η_0 are initial values and $\Phi(\sigma) = \Phi(\sigma_1,\sigma_2)$.

It is clear that the loci are found as the intersections between cylindrical surfaces whose generators are parallel to the σ_1 and σ_2 axes. The point O is a stable special point. Then the projection of a locus on the σ_2,η plane never depends on σ_1, while σ_1 defines in the σ_2,η plane only the point where $\Phi(\sigma_1,\sigma_2)$ intersects it. The projection on the $\sigma_1\eta$ plane never depends on σ_2, while σ_2 defines in the σ_1,η plane only the point where $\Phi(\sigma_1,\sigma_2)$ intersects it.

Hence, our problem becomes to study the motions of the projections on the σ_1,η and σ_2,η planes simultaneously, because these projections represent fully all possible types of motion caused by the various initial deviations in the σ_1,σ_2,η space.

The Phase Planes for the Loop

We consider the σ_1,η and σ_2,η in the σ_1,σ_2,η space as the phase planes for the loop. We represent them as planes of many sheets, because it is convenient to project each of our rectangular parallelepipeds on a different sheet. The projections of the loci on the σ_1,η and σ_2,η planes are given by (15)-(18). Each of the σ_1,η and σ_2,η planes consists of six sheets. The lines of intersection are $\sigma_1 = \epsilon_1$, $\sigma_1 = \epsilon_2$, $\eta = 0$ on the σ_1,η plane, and $\sigma_2 = \epsilon_1$, $\sigma_2 = \epsilon_2$, $\eta = 0$ on the σ_2,η plane. The σ_1,η and σ_2,η planes have bounds $\sigma_1 = 0$, $\sigma_2 = 0$, $\eta = \pm 1$, $\sigma_1 = \gamma$, $\sigma_2 = \gamma$.

Figure 7 shows the σ_1,η and σ_2,η phase planes, and indicates how the sheets are denoted. Each plane has two projections of the special point O, which is the equilibrium point for the system. The other special points lie outside the phase planes, on account of the restrictions that have been applied. Some of the volumes in σ_1,σ_2,η space overlap (Fig. 6), so sheets I, III, IV and VI in the σ_1,η plane, and I', III', IV' and VI' in the σ_2,η plane, have identical bounds, and are exactly superimposed.

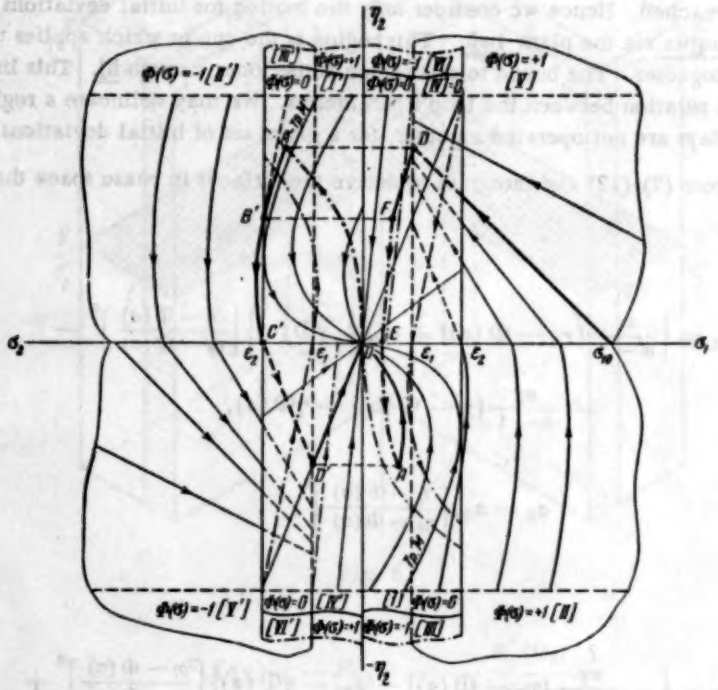

Fig. 7.

The condition that the two relays do not operate together limits the parameter space available. The region of initial deviations for which this condition must be complied with must be given when the bounds are to be found. Here we take this region to be that bounded on sheets V and III' by the loci that pass through $\sigma_1 = \gamma$, $\eta = 0$ and $\sigma_2 = \epsilon_1$, $\eta = 0$. The equations for these loci give us the relation between the parameters that defines the bounds in the following form:

$$\left(\frac{\gamma}{a-1}+\varepsilon_1\right)\left(\frac{\epsilon_1}{\gamma}\right)^{a/b}-\frac{a\gamma}{a-1}\left(\frac{\epsilon_1}{\gamma}\right)^{1/b}-\varepsilon_2+\gamma=0. \tag{19}$$

We may analyze the σ_1,η and σ_2,η phase planes to find two main regions of initial deviations; to these will correspond different types of motion. In the first, the tendency is always to approach the point of rest. Within this region the image point always approaches the rest point; the transient may or may not show an overshoot. There will be two types of motion within this region.

1. In the first type the point lies either in $\eta < 0$ or in $\eta > 0$, and reaches O without passing outside its initial half-plane. On the σ_1 or σ_2 axis there is a bound R to the initial deviations σ, for which such motion can occur.

Figure 7 shows that this motion will occur in the top half-plane if the change-over point A' on σ_2,η lies within the region bounded by the locus M' that touches the line $\sigma_2 = \epsilon_2$. To find R on the σ_1 axis it is, however, better to consider the points where M' and sheet I' (which passes through A') intersect the η axis (η_m and η^*). The bound R corresponds to $\eta_m = \eta^*$. We use the equations for the loci on sheets V, III' and I' to find an expression for R:

$$\frac{\epsilon_2}{a-1}\left(\frac{\gamma}{\epsilon_2}\right)^a = \frac{\frac{\gamma}{a-1}\left(\sigma_{20}+\frac{\gamma}{a-1}\right)\left(\frac{\epsilon_1}{R}\right)^{a/b}}{\left[1-\left(\frac{\epsilon_1}{R}\right)^{1/b}\right]^a}. \tag{20}$$

R is found on the σ_2 axis by putting σ_{10} instead of σ_{20} in (20).

2. In the second type the point lies at first either in $\eta > 0$ or in $\eta < 0$, and passes to the other half-plane before approaching O. On the σ_1 or σ_2 axis there is a bound S to the initial deviations for which such motion exists. Figure 7 shows that, if such motion starts in the top half-plane, we must have the point A in the σ_1,η plane to lie in a region bounded by the locus M, which touches the line $\sigma_1 = \epsilon_1$. Bound S on σ_1 will correspond to R on σ_2. We use the equations for the loci on the upper half-plane, with (20), to get this expression for S:

$$\left(\sigma_{20}+\frac{\gamma}{a-1}\right)\left(\frac{\epsilon_1}{S}\right)^{a/b}\frac{-\gamma}{a-1} = \left(\epsilon_2-\frac{a\gamma}{a-1}f\right)\left[\frac{1-\left(\frac{\epsilon_1}{S}\right)^{1/b}}{f}\right]^a, \tag{21}$$

$$\frac{\epsilon_2}{a-1}\left(\frac{\gamma}{\epsilon_2}\right)^a = \frac{\frac{\gamma}{a-1} - H\left[\frac{\gamma}{a-1}+\epsilon_1\left\{\frac{f}{(f+1)\left[1-\left(\frac{\epsilon_1}{S}\right)^{1/b}\right]}\right\}^b\right]}{\left\{1-\left[\frac{\epsilon_1}{\left(\epsilon_2-\frac{a\gamma}{a-1}f-\frac{\gamma}{a-1}\right)\left(\frac{1}{f+1}\right)^a+\frac{\gamma}{a-1}}\right]^{1/b}\right\}^a}$$

where

$$H=\left[\frac{\epsilon_1}{\left(\epsilon_2-\frac{a\gamma}{a-1}f-\frac{\gamma}{a-1}\right)\left(\frac{1}{f+1}\right)^a+\frac{\gamma}{a-1}}\right]^{a/b}, \tag{22}$$

f is the ordinate of the point F.

The bound S divides off the region within which the rest point is approached. This second type of motion may not exist for some ratios between the parameters. Then S vanishes, and the bound R takes over its function.

The second region is the one in which the transient is oscillatory. The motion of a point initially in $\eta > 0$ or $\eta < 0$ is then such as to take the point in turn through all six sheets, and back to the initial sheet. The loci may spiral inward, so that the rest point is eventually reached, but closed loops may exist. We shall study the motion in this region by point-transform methods.

The Point Transform

To seek limiting cycles and to study loci in the σ_1,η and σ_2,η planes we examine the point transforms of a straight line into a straight line on each of the planes. It is best to see how the points on the σ_1 axis transform to points on the same axis in the σ_1,η plane. The point transform for any point is made up of two correspondence functions. For σ_1 these functions are A, which defines the transform from points on the σ_1 axis with $\sigma_1 > \epsilon_1$ to ones with $\epsilon_1 > \sigma_1 > 0$ (top half-plane), and B, which defines the transform from points on the σ_1 axis with $\epsilon_1 > \sigma_1 > 0$ to ones with $\sigma_1 > \epsilon_1$. Function C defines the transform from points on the σ_2 axis with $\epsilon_1 > \sigma_2 > 0$ to ones with $\sigma_2 > \epsilon_1$, while D defines the transform from points on the σ_2 axis with $\sigma_2 > \epsilon_1$ to ones with $\epsilon_1 > \sigma_2 > 0$. The two phase planes are the same, so A is the same as D, and B is the same as C.

The point-transforms diagram will consist of two parts, each of which defines the transform for the coordinates that correspond to itself. The final and initial σ_1 and σ_2 values are related, so the diagram will be a space, and the functions will define surfaces. We must consider both parts in order to follow the motion of the image points as the number of transfers increases. The fixed points can be found by considering one coordinate only, because the functions are the same for both parts. We consider the point transform for σ_1 below, and examine the behavior of A and B in general terms.

Half-plane $\eta > 0$. Let the coordinates at the start be $\sigma_1 = u$, $\eta = 0$, $\sigma_2 = w$. We consider A, so $u > \epsilon_1$,

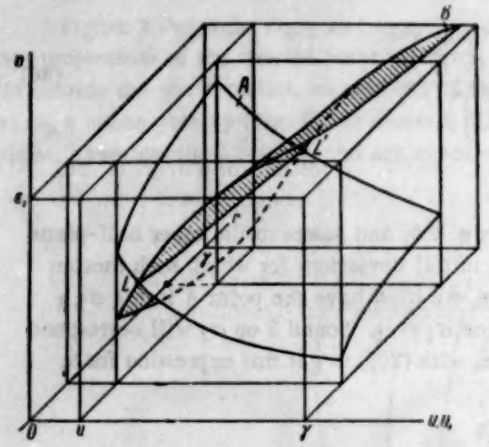

Fig. 8.

$0 < w < \varepsilon_1$. Now take some time $t^* = t_1 + t_2 + t_3$, where t_1, t_2 and t_3 are the times spent by the image point in sheets V, IV and VI, when the coordinates are $\sigma_1 = v$, $\eta = 0$, $\sigma_2 = r$.

We use the solutions to (8) and (9) for t_1, t_2 and t_3, and eliminate from these t_1, t_2, t_3, $\underline{w}$, and intermediate values of certain coordinates; we then have relations between $\underline{u}$, $\underline{v}$ and $\underline{r}$:

$$v = \left(\frac{f}{1+f}\right)^b (u^{1/b} - \varepsilon_1^{1/b})^{-b} u\varepsilon_1, \tag{23}$$

$$r = \left(\varepsilon_2 + \frac{a\gamma}{1-a} f + \frac{\gamma}{1-a}\right)(1+f)^{-a} - \frac{\gamma}{1-a}. \tag{24}$$

Half-plane $\eta < 0$. Let the coordinates at the start be $\sigma_1 = v$, $\sigma_2 = r$, $\eta = 0$. Consider some time $t^{**} = t_4 + t_5 + t_6$, where t_4, t_5 and t_6 are the times spent by the image point in sheets III, I and II, when the coordinates are $\sigma_1 = u_1$, $\sigma_2 = w_1$, $\eta = 0$. We use the solutions to (7) and (9) for t_4, t_5 and t_6, and eliminate w_1, t_4, t_5 and t_6, and certain intermediate values of coordinates; we get expressions relating u_1, $\underline{v}$ and $\underline{r}$:

$$u_1 = \left(\varepsilon_2 - \frac{a\gamma}{1-a}\varphi + \frac{\gamma}{1-a}\right)(1-\varphi)^{-a} - \frac{\gamma}{1-a}, \tag{25}$$

$$v = \left(\varepsilon_2 - \frac{a\gamma}{1-a}\varphi\right)\left[1 - \left(\frac{r}{\varepsilon_1}\right)^{1/b}\right]^a \varphi^{-a} - \frac{\gamma}{1-a}\left[\left(\frac{r}{\varepsilon_1}\right)^{a/b} - 1\right], \tag{26}$$

where φ is the ordinate of the point where the trace of sheet I meets the line $\sigma_1 = \varepsilon_2$. The Appendix examines, in a general way, the behavior of A [which is defined by (23) and (24)] and of B [which is defined by (25) and (26)] for the case $a > 1$, $b > a$.

Figure 8 shows part of the point-transforms diagram for σ_1 in $\underline{u}$, $\underline{v}$, $\underline{r}$ coordinates. Here A (a surface) is shown for $\partial^2 v/\partial r^2 < 0$, and B for $\partial^2 v/\partial r^2 > 0$. The second part of the diagram relates to σ_2, and is in $\underline{u}$, $\underline{w}$, $\underline{r}$ coordinates. The A and B surfaces meet in one line LL' only. Figure 8 also shows that, with this disposition of the surfaces, $\underline{u}$ will change as the number of transfers increases in such a way as to approach LL' (whatever $\underline{r}$), and hence that the fixed points defining the limits to the cycle must lie on LL'. Now limits can exist on both planes only if $u = r$, so the point corresponding to the limit will be the point where LL' meets the plane $u = r$. Consider the plane $v = \text{const}$ in order to see how LL' lies, with respect to the plane $u = r$. This latter plane will give a trace of the form $u = r$ on the $r = \text{const}$ plane, whereas LL' will give a projection ll', whose equation is given by (23)-(26) as

$$\left(\frac{1+f}{f}\right)^{-b}(u^{1/b} - \varepsilon_1^{1/b})^{-b} u\varepsilon_1 = \left(\varepsilon_2 - \frac{\gamma a}{1-a}\varphi\right)\varphi^{-a}\left[1 - \left(\frac{r}{\varepsilon_1}\right)^{1/b}\right]^a - \frac{\gamma}{1-a}\left[\left(\frac{r}{\varepsilon_1}\right)^{a/b} - 1\right], \tag{27}$$

$$r = \left(\varepsilon_2 + \frac{a\gamma}{1-a} f + \frac{\gamma}{1-a}\right)(1+f)^{-a} - \frac{\gamma}{a-1}, \tag{28}$$

$$u = \left(\varepsilon_2 - \frac{a\gamma}{1-a}\varphi + \frac{\gamma}{1-a}\right)(1-\varphi)^{-a} - 1\frac{\gamma}{1-a}. \tag{29}$$

Now the equation for ll' is complex, so we cannot consider its behavior fully, but some general deductions can be made. The derivative du/dr for ll' is given by

$$\frac{du}{dr} = -\frac{\left(\frac{1}{\varepsilon_1}\right)^{\frac{a}{b}} \frac{a}{br} K - \frac{b\varepsilon_1 u f^{b-1}(1+f)^{a-b}}{a(\gamma f - \varepsilon_2)(u^{\frac{1}{b}} - \varepsilon_1^{1/b})^b}}{-\left(\frac{1}{\varepsilon_1}\right)^b \left(\varepsilon_1^{\frac{1}{b}} - r^{\frac{1}{b}}\right)^a \left(\frac{1-\varphi}{\varphi}\right)^{a+1} + \left(\frac{f}{f+1}\right)^b \left(u^{\frac{1}{b}} - \varepsilon_1^{\frac{1}{b}}\right)^{-b-1} \varepsilon_1^{1+\frac{1}{b}}},$$

where

$$K = -\left(\varepsilon_2 - \frac{a\gamma}{1-a}\varphi\right)\varphi^{-a} r^{\frac{1}{b}} \left(\varepsilon_1^{\frac{1}{b}} - r^{\frac{1}{b}}\right)^{a-1} + \frac{\gamma}{a-1} r^{\frac{a}{b}}.$$

Now du/dr is positive for all r, and when $r = r_c$ we have $du/dr = \infty$. The expression for d^2u/dr^2 is very complex, and even the sign of the derivative cannot be found from it. We may say, though, that if r is close to r_c, we must have d^2u/dr^2 negative, because du/dr, while positive can be less only than $+\infty$.

The phase-plane diagrams also show that the end of ll' (i.e., l') must always lie above the straight line $u = r$, since the bound to the point transform, r^*, corresponds to $f = 1$, and hence, from the transform for A, to $u = \infty$. But $r = r^*$ gives us for the transform of B a value of u_1 less than r^*. The shapes of the A and B surfaces show that ll' may, in general, have several inflections.

Bifurcation Relations for the Parameters

The following disposition of ll' relative to the line $u = r$ can occur.

1. Line ll' lies above $u = r$ (Fig. 9). The motion is then stable for all initial deviations.

2. Line ll' cuts $u = r$, but does not reach the bound $r = r_c$ (Fig. 10). There can then be any even number of intersections, to which will correspond stable and unstable limiting cycles. Stable motion will still occur for some initial deviations. The dividing line between the first and second cases corresponds to ll' touching $u = r$. Then we must have $u = r$ and $du/dr = 1$. We use these conditions, with (27)-(30), to give the bifurcation relationships as

$$r\varepsilon_1\left(\frac{f}{1+f}\right)^b (r^{1/b} - \varepsilon_1^{1/b})^{-b} = \left(\frac{1}{\varepsilon_1}\right)^{a/b}\left[\left(\varepsilon_2 + \frac{a\gamma}{1-a} f\right)(r^{1/b} - \varepsilon_1^{1/b})^a f^{-a} - \right.$$
$$\left. - \frac{\gamma}{a-1}(r^{a/b} - \varepsilon_1^{a/b})\right], \tag{31}$$

$$\frac{b\varepsilon_1 r f^{b-1}(1+f)^{a-b}}{a(\gamma f - \varepsilon_2)(r^{1/b} - \varepsilon_1^{1/b})^b} - \left(\frac{1}{\varepsilon_1}\right)^{a/b} \frac{a}{br}\left[\frac{r^{1/b}[\varepsilon^2 - a\gamma f/(1-a)]}{f^a(r^{1/b} - \varepsilon_1^{1/b})^{1-a}} + \frac{\gamma}{a-1} r^{a/b}\right] = 1, \tag{32}$$
$$\left(\frac{1}{\varepsilon_1}\right)^{a/b}(r^{1/b} - \varepsilon_1^{1/b})^a\left(\frac{f}{1+f}\right)^{-a-1} + \left(\frac{f}{1+f}\right)^b (r^{1/b} - \varepsilon_1^{1/b})^{-b-1}\varepsilon_1^{1+1/b}$$

$$r = \left(\varepsilon_2 + \frac{a\gamma}{1-a} f + \frac{\gamma}{1-a}\right)(1+f)^{-a} - \frac{\gamma}{1-a}. \tag{33}$$

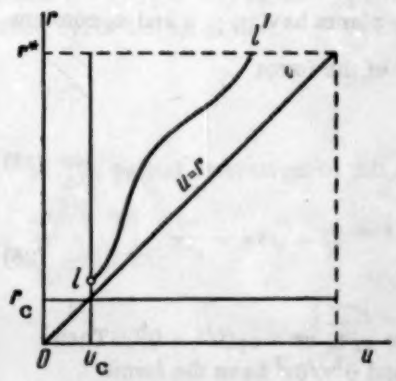

Fig. 9.

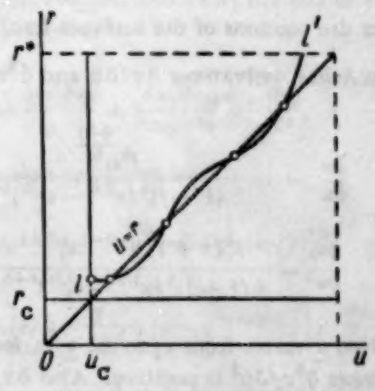

Fig. 10.

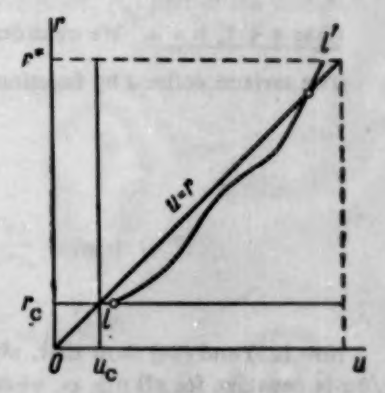

Fig. 11.

Now both conditions, $u = r$ and $du/dr = 1$, are allowed for in (32), whereas only $u = r$ is allowed for in (31), so the roots of (32) are also roots of (31). The two equations give the critical relation between the parameters when they have a common root. If (31) has extra roots, ll' and $u = r$ will cut as well as be tangential. If (32) loses or gains a root, or (31) two, we get a change in the type of motion, and may find a bifurcation relation.

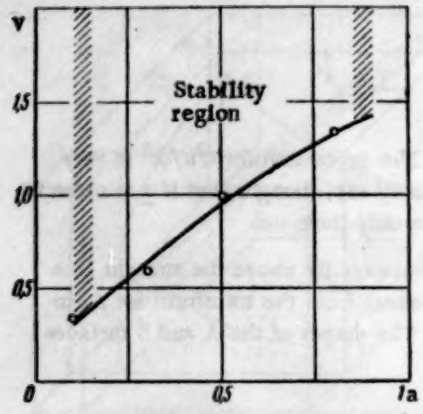

Fig. 12.

3. The line ll' cuts u = r, and ll' passes through the entire region $r_c \leq r \leq r^*$ (Fig. 11). There must then be at least one intersection of ll' with u = r. The A and B surfaces considered here are so placed that this point corresponds to a stable limit. All deviations within the second region will then give unstable motion.

The boundary between the second and third cases occurs when l lies at the point $u = r_c$. This point will correspond to cycles that include parts of loci M and M'. The coordinates and equations for the loci give us the bifurcation relation for the second and third types in the form

$$\gamma - \varepsilon_2\left[\frac{\gamma(1-G)}{\varepsilon_2}\right]^a = \left\{\gamma + \varepsilon_1(a-1)\left[\frac{\varepsilon_2}{(\gamma+\varepsilon_2)(1-G)}\right]^b\right\}G^a, \quad (34)$$

where

$$G = \left[\frac{\varepsilon_1(a-1)}{\gamma - (\varepsilon_2+\gamma)^{1-a}(\gamma)^a}\right]^{1/b}.$$

Hence our study of the possible positions of ll' relative to the line u = r on the v = const plane has given us equations from which we may delineate areas in phase space with differing structures within the region where the transients are oscillatory. There are three main regions in phase space. In the first, the motion is stable, in the second, the motion may or may not be stable, and in the third, we get hunting. The boundary between the first and second regions is the stability boundary, and is defined by (31) and (32). Equation (34) defines the bounds to the third region.

The loop must always be stable, and so in any real case we have to find only the stability limit, i.e., to find only that relation between the parameters for which (31) and (32) have a common root.

Figure 12 shows the stability bounds in the a,b plane for a loop with $\gamma = 1$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.2$. The projection of ll' for a from 0.1 to 0.9 is of a shape such that only one contact between it and the line u = r is possible in the v = const plane; hence (31)-(33) give us the relation that defines the stability limit.

APPENDIX

We examine the shapes of the surfaces defined by (23)-(26). We assume that $a < 1$, $b > a$, which is so for the loop in an electrical autopilot.

Case $a < 1$, $b > a$. We consider the sections of the surfaces formed by planes having r, u and u_1 constant.

The surface defined by function A has derivatives $\partial v/\partial u$ and $\partial^2 v/\partial u^2$ of the forms

$$\frac{\partial v}{\partial u} = -\frac{f^b \varepsilon_1^{\frac{b+1}{b}}}{(1+f)^b (u^{1/b} - \varepsilon_1^{1/b})^{1+b}}, \quad (35)$$

$$\frac{\partial^2 v}{\partial u^2} = \frac{f^b (b+1) u^{1/b-1} \varepsilon_1^{1/b+1}}{b(1+f)^b (u^{1/b} - \varepsilon_1^{1/b})^{b+2}}. \quad (36)$$

Now (23) and (24) show that, when u varies from ϵ_1 to ∞, v varies from ϵ_1 to $v^* = \epsilon_1(f/1+f)^b$. Then $\partial v/\partial u$ is negative for all $u > \epsilon_1$, whereas $\partial^2 u/\partial u^2$ is positive. Also $\partial v/\partial r$ and $\partial^2 v/\partial r^2$ have the forms

$$\frac{\partial v}{\partial r} = \frac{u\varepsilon_1 b f^{b-1}(1+f)^{a-b}}{a(\gamma f - \varepsilon_2)(u^{1/b} - \varepsilon_1^{1/b})^b}, \quad (37)$$

$$\frac{\partial^2 v}{\partial r^2} = \frac{u\varepsilon_1 b f^{b-2}(1+f)^{2a-b}}{a^2(u^{1/b} - \varepsilon_1^{1/b})^b} \frac{[b-1-f(1-a)](\gamma f - \varepsilon_2) - \gamma f^2 - \gamma f}{(\gamma f - \varepsilon_2)^3}. \quad (38)$$

These expressions show that, if $r > r_c$, i.e., if $f > \epsilon_2/\gamma$, $\partial v/\partial r$ is positive. The latter is $+\infty$ if $r = r_c$. Then r_c is the bound to the point transform, and corresponds to $f_c = \epsilon_2/\gamma$. Now $\partial^2 v/\partial r^2$ is negative when $b \leq 1$ for all $\underline{r}$. If $b > 1$, it is possible for $\partial^2 v/\partial r^2$ to change sign once or twice, since $[b - 1 - f(1 - a)](\gamma f - \epsilon_2) - \gamma f^2 - \gamma f$ can become zero for two values of $\underline{f}$.

Our study of surface A thus shows that this surface intersects a plane r = const to give a falling curve for all $\underline{r}$ such that $r_c < r < r^*$, and that $\underline{v}$ will tend to a definite limit if $u \to \infty$. The sections in a plane u = const depend on $\underline{b}$, and can be of three types.

Let us study surface B (function B). The expressions for $\partial v/\partial u_1$ and $\partial^2 v/\partial u_1^2$ are

$$\frac{\partial v}{\partial u_1} = -[1 - (r/\epsilon_1)^{1/b}]^a \left(\frac{1-\varphi}{\varphi}\right)^{a+1}, \tag{39}$$

$$\frac{\partial^2 v}{\partial u_1^2} = \frac{(a+1)(1-\varphi)^{2a+1}[1-(r/\epsilon_1)]^a}{a\varphi^{a+2}(\gamma\varphi - \epsilon_2)}. \tag{40}$$

Now function B has a bound $u_1 = u_{1c}$, which corresponds to $\varphi_c = -\epsilon_2/\gamma$.

Then (25) and (26) give us that u_1 changes from u_{1c} to $u^*_1 = r^*$ as φ varies from φ_c to -1, while $\underline{v}$ varies from v^* to ϵ_1, where

$$v^* = \frac{\epsilon_1}{1-a}\left[\left(\frac{r}{\epsilon_1}\right)^{1/b} - 1\right]^a \left(\frac{\epsilon_2}{\gamma}\right)^{-a} - \frac{\gamma}{1-a}\left[\left(\frac{r}{\epsilon_1}\right)^{a/b} - 1\right]. \tag{41}$$

But $\partial v/\partial u_1$ is positive if $\varphi < \varphi_c$. However, $\partial^2 v/\partial u_1^2$ is negative when $\varphi \leq \varphi_c$.

Let us examine the section of surface B by planes with u_1 = const. The expressions for $\partial v/\partial r$ and $\partial^2 v/\partial r^2$ can be derived from (26), but they are very complex, and the signs of the derivatives cannot be found. We use the equations for the loci in the lower half-plane and the solutions to (7) and (9) to relate $\underline{v}$ and $\underline{r}$ in an expanded form:

$$v = \left(\frac{a\gamma}{a-1}\eta_4 + \sigma_{14} + \frac{\gamma}{a-1}\right)(\eta_4 + 1)^{-a} - \frac{\gamma}{a-1}, \quad \eta_4 = \left(\frac{\epsilon_1}{r}\right)^{1/b} - 1,$$

$$\sigma_{14} = \left(\epsilon_2 + \frac{a\gamma}{a-1}\varphi\right)e^{ct_5} - \frac{a\gamma}{a-1}\varphi e^{ht_5}, \quad \eta_4 = \varphi e^{ht_5},$$

where $c = 1/T_1$, $h = 1/T_3$, and η_4 and σ_{14} are the coordinates at the end of the fourth (t_4) part of the motion. Then $\partial v/\partial r$ can be put as

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial \eta_4}\frac{\partial \eta_4}{\partial r} + \frac{\partial v}{\partial \sigma_{14}}\frac{\partial \sigma_{14}}{\partial t_5}\frac{\partial t_5}{\partial \eta_4}\frac{\partial \eta_4}{\partial r}.$$

The partial derivatives in this expression are given by

$$\frac{\partial v}{\partial \eta_4} = a(\eta_4 + 1)^{-a-1}(-\gamma\eta_4 - \sigma_{14}), \quad \frac{\partial \eta_4}{\partial r} = -\frac{1}{br}(\epsilon_1/r)^{1/b}, \quad \frac{\partial v}{\partial \sigma_{14}} = (\eta_4 + 1)^{-a},$$

$$\frac{\partial \sigma_{14}}{\partial t_5} = ce^{ct_5}\left\{\epsilon_2 + \frac{\gamma}{a-1}\varphi[a - e^{(1-a)ht_5}]\right\}, \quad \frac{\partial t_5}{\partial \eta_4} = \frac{1}{h\varphi}e^{-ht_5}.$$

These formulas show that $\partial \eta_4/\partial r$ is negative, that $\partial v/\partial \eta_4$ and $\partial v/\partial \sigma_{14}$ are positive when $-1 < \eta_4 < -\epsilon_2/\gamma$ that $\partial \sigma_{14}/\partial t_5$ is negative when $\varphi < -\epsilon_2/\gamma$ and $t_5 > 0$, and that $\partial t_5/\partial \eta_4$ is also negative. Hence $\partial v/\partial r$, which can be put as

$$\frac{\partial v}{\partial r} = s = -\frac{a}{b\epsilon_1}(1 + \eta_4)^{b+1-a}\left[\frac{-\sigma_{14} - \gamma\eta_4}{\eta_4 + 1} + \frac{\frac{a\gamma}{a-1}\varphi + \epsilon_2}{\eta_4}\left(\frac{\eta_4}{\varphi}\right)^a - \frac{\gamma}{a-1}\right], \tag{42}$$

is negative for all the $\underline{r}$ and $\underline{u}$ within the set range.

Now $\partial^2 v/\partial r^2$ is given by

$$\frac{\partial^2 v}{\partial r^2} = \frac{\partial s}{\partial r} = \frac{a(b-a)}{b^2 c_1^2}(1+\eta_4)^{2b+1-a} \times$$

$$\times \left[\frac{-\sigma_{14} - \gamma\eta_4}{\eta_4 + 1} + \frac{\frac{a\gamma}{a-1}\varphi + c_2}{\eta_4}\left(\frac{\eta_4}{\varphi}\right)^a \left(1 + \frac{a-1}{(b-a)\eta_4}\right) - \frac{\gamma}{a-1}\right]. \qquad (43)$$

The expression in square brackets in (43) differs from the similar one in (42) in that the negative term is multiplied by a quantity greater than unity which decreases as $\underline{r}$ increases. Hence, $\partial^2 v/\partial r^2$ can be either positive or negative, and the change between the two may occur once. The reason is that $\partial v/\partial r$ is negative for all $\underline{r}$.

Our study of surface B shows that the sections by planes with r = const are rising curves, and that when $u_1 \to \infty$, $r \to \infty$. The sections given by planes with u_1 = const can take three forms; which form is found depends on $\underline{u}$, $\underline{r}$ and the relation between the parameters.

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COMPENSATION OF A CONTINUOUS AUTOMATIC CONTROL SYSTEM BY MEANS OF A DELAY ELEMENT FILTER

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Methods are considered for the design of a delay-element filter for compensating a continuous automatic control system. The filter parameters are determined for a few simple cases.

For some time past papers have appeared in the literature, dealing with the use of a delay-element filter (DEF) for improving the dynamic quality of continuous open-loop and closed-loop systems. When compared to an ordinary compensating network a DEF has several advantages: flexibility in the selection of its parameters and in adjustment, convenience of synthesizing the filter from a given pulse characteristic, etc. These advantages make filters of this type particularly convenient for the compensation of systems and for use in simulation [1-6]. The characteristics of the DEF also permit one to design a system which attains a steady-state mode in a finite time interval [1, 2, 3, 7] and to compensate the delay in an automatic control system [8].

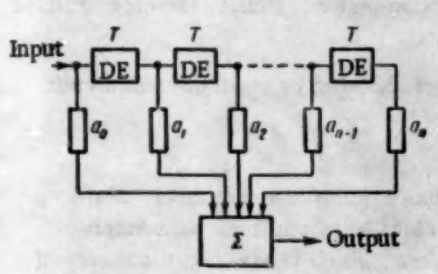


Fig. 1.

In this paper, the use of a DEF as a compensating device in continuous automatic control systems is considered.

The DEF shown in block-diagram form in Fig. 1 has a transfer function of the form

$$D(p) = a_0 + a_1 e^{-pT} + a_2 e^{-2pT} + \ldots + a_n e^{-npT},$$

where T is the delay time of each delay element and n is the order of the filter, characterizing the number of delay elements.

The problem of compensating a continuous automatic control system by means of a DEF consists of the determination of the order of the filter and the values of its parameters for which the system takes on the required properties. It is preferable to choose the order of the filter as low as possible, and, at the same time, its parameters must be optimum in a particular sense.

Determination of the Order of a DEF

Let us examine the automatic control system, the schematic of which is shown in Fig. 2, where the expression

$$K(p) = \frac{1}{b_0 + b_1 p + b_2 p^2 + \ldots + b_m p^m}$$

represents the transfer function of the permanent part of the system, while D(p) represents the transfer function of the compensating DEF. A considerable number of networks corresponds to the form of transfer function given

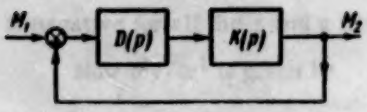


Fig. 2.

by K(p). Moreover, the compensation of a system by such a circuit represents a more difficult case then the compensation of a system by a circuit having a polynomial in p in the numerator of the function K(p).

Ideal compensation of a system requires that for a static system the relationship

$$D(p)K(p) = C \tag{1}$$

be satisfied, while for an astatic system the relationship

$$D(p)K(p) = C/p \tag{2}$$

be satisfied, where C is a constant which should be chosen as large as possible. Because of the limitation of permissible power and the existence of saturation in practical systems, condition (1) or (2) can be satisfied over a definite frequency range only.

We know from the papers which dealt with the case of the compensation of an open-loop system [1, 2] that condition (1) can be satisfied over the range of frequencies of interest to us by making use of a DEF. In this case the sufficient order of the filter is equal to the order of the system being compensated.

It follows from what has been said that to compensate a static, automatic control system one may select the order of the filter equal to the order of the open-loop system, while for an astatic system the order of the filter will be one lower.

In a real system, the time constants of the fixed part of the system do not all have the same effect on the properties of the system. It is obvious that to ensure the required properties of the system it suffices to take into account those time constants which have the greatest effect on the system's properties. Hence, the order of filter can be chosen tentatively to be equal to the number of these time constants.

Having chosen the order of the filter, we will examine the question of choosing its optimum parameters.

Optimum Parameters of a DEF

The problem of obtaining optimum parameters is equivalent to the basic problem of the theory of the approximation of functions in mathematics. Different criteria exist for the selection of optimum parameters of controlled systems. We will examine here the method of selecting parameters, which is based on the theory of the approximation of a function by the method of the expansion of the frequency characteristic of a control system into a Taylor series. The parameters are called optimum when the frequency characteristic of the compensated system deviates by the least amount from its value when $\omega = 0$.

The method of expanding a frequency characteristic into a series was first used by Braude in the synthesis of reproducing systems [9]. The use of this method for the choice of optimum parameters of control systems was described, for example, in papers [10, 11]. Below we will examine the application of this method of choosing parameters to our case.

The transfer function of the closed-loop control system shown in Fig. 2 is equal to

$$K_c(p) = \frac{K(p)D(p)}{1 + K(p)D(p)} = \frac{M(p)}{\Delta(p)}, \tag{3}$$

where

$$M(p) = a_0 + a_1 e^{-pT} + \cdots + a_n e^{-npT},$$

$$\Delta(p) = b_0 + b_1 p + \cdots + b_m p^m + a_0 + a_1 e^{-pT} + \cdots + a_n e^{-npT}.$$

If we expand the square of the modulus of the frequency characteristic $|K_c(j\omega)|^2 = \lambda(\omega)$ into a Taylor series about the point $\omega = 0$

$$\lambda(\omega) = \lambda(0) + \lambda''(0)\frac{\omega^2}{2!} + \lambda^{(4)}(\,)\frac{\omega^4}{4!} + \cdots \tag{4}$$

(the odd derivatives are identically equal to zero) we obtain the conditions for optima [10]:

$$\lambda''(0) = \lambda^{(4)}(0) = \cdots = \lambda^{(2k)}(0) = \cdots = 0. \tag{5}$$

In this case the frequency characteristic $\lambda(\omega)$ has its smallest deviation from $\lambda(0)$. The problem of choosing optimum parameters consists of finding expressions for the conditions for optima (5) from the filter parameters, appearing in Eq. (3), and then determining their values.

We find an expression for $\lambda(\omega)$ from Eq. (3):

$$\lambda(\omega) = |K_c(j\omega)|^2 = \frac{M(j\omega)M(-j\omega)}{\Delta(j\omega)\Delta(-j\omega)}.$$

Expanding the numerator $M(j\omega)M(-j\omega)$ into a series (the odd terms are identically equal to zero) we obtain

$$M(j\omega)M(-j\omega) = M_0 + M_2\omega^2 + M_4\omega^4 + \cdots,$$

where

$$M_{2l} = \frac{1}{(2l)!}\frac{d^{2l}[M(j\omega)M(-j\omega)]}{d\omega^{2l}}\Big|_{\omega=0} = \frac{1}{(2l)!}\sum_{k=0}^{2l}\binom{2l}{k}\frac{d^k M(j\omega)}{d\omega^k}\frac{d^{2l-k}M(-j\omega)}{d\omega^{2l-k}}\Big|_{\omega=0} =$$

$$= \sum_{k=0}^{2l}\frac{1}{k!(2l-k)!}(-1)^k(j)^{2l}\frac{d^k M(j\omega)}{d(j\omega)^k}\frac{d^{2l-k}M(-j\omega)}{d(-j\omega)^{2l-k}}\Big|_{\omega=0} \quad (l=0,1,2,\ldots).$$

Since here the differentiation does not depend on the sign of ω then, after replacing $j\omega$ and $-j\omega$ by p, we obtain

$$M_{2l} = \sum_{k=0}^{2l}(-1)^{k+l}\frac{M^{(k)}(p)M^{(2l-k)}(p)}{k!(2l-k)!}\Big|_{p=0} \quad (l=0,1,2,\ldots). \tag{6}$$

Analogously, for the denominator of expression (4) we have

$$\Delta(j\omega)\Delta(-j\omega) = \Delta_0 + \Delta_2\omega^2 + \Delta_4\omega^4 + \cdots,$$

where

$$\Delta_{2l} = \sum_{k=0}^{2l}(-1)^{k+l}\frac{\Delta^{(k)}(p)\Delta^{(2l-k)}(p)}{k!(2l-k)!}\Big|_{p=0} \quad (l=0,1,2,\ldots). \tag{7}$$

The expression for $\lambda(\omega)$ can now be written in the following form:

$$\lambda(\omega) = \frac{M_0 + M_2\omega^2 + M_4\omega^4 + \cdots}{\Delta_0 + \Delta_2\omega^2 + \Delta_4\omega^4 + \cdots}. \tag{8}$$

It should be noted that relationships (6) and (7) permit one to calculate M_{2l} and Δ_{2l} directly from the transfer function $K_c(p)$ without having to find the square of the modulus of $\lambda(\omega)$.

Let us examine the expression for the deviation of the frequency characteristic $\lambda(\omega)$ from $\lambda(0)$:

$$\lambda(\omega) - \lambda(0) = \frac{(M_2\Delta_0 - M_0\Delta_2)\omega^2 + (M_4\Delta_0 - M_0\Delta_4)\omega^4 + \cdots}{\Delta_0(\Delta_0 + \Delta_2\omega^2 + \Delta_4\omega^4 + \cdots)}. \tag{9}$$

A comparison of expressions (4) and (9) shows that the quantity $\lambda''(0)$ will be equal to zero if the condition $M_2/\Delta_2 = M_0/\Delta_0$ is satisfied.

Analogously, the derivative $\lambda^{(4)}(0)$ will be equal to zero if the conditions $M_4/\Delta_4 = M_2/\Delta_2 = M_0/\Delta_0$ are satisfied, etc. Hence, the conditions for optimum filter parameters can be written in the following manner:

$$\frac{M_{2l}}{\Delta_{2l}} = \frac{M_0}{\Delta_0} \quad (l = 1, 2, \ldots). \tag{10}$$

In the case of an astatic system ($M_0 = \Delta_0$) expression (10) takes on the form:

$$M_{2l} = \Delta_{2l} \qquad (l = 1, 2, \ldots). \tag{11}$$

In determining the optimum parameters ($a_0, a_1, \ldots, a_n, T$) of a DEF from Eqs. (10) or (11) one must use the $n + 2$ equations which correspond to the $n + 2$ first (even) derivatives of the square of $\lambda(\omega)$ at the point $\omega = 0$ being equal to zero.

In this way, the selection of the optimum parameters of a DEF consists of the following: values of M_{2l} and Δ_{2l} are determined from expressions (6) and (7) under the conditions $l = 0, 1, \ldots, n, n + 1, n + 2$, where n is the order of the chosen filter; all the ratios M_{2l}/Δ_{2l} are equated to the value of M_0/Δ_0 (for a static system) or to unity (for an astatic system); all the desired parameters of the filter are determined from these equations (conditions for optima).

In determining the filter parameters one can take into account other restrictions, such as the required values of the error coefficients, limiting values of several filter coefficients a_l, etc.

The transient characteristic of the compensated system must be checked with the chosen filter parameters. If the characteristic does not satisfy the required properties of the system, then it is necessary to raise the order of the filter and repeat the calculations.

Comparison of Ordinary Filters with a DEF

The compensation of a control system by a DEF is achieved by introducing, beside a signal proportional to the error at a given instant of time, signals proportional to the errors at previous instants. This, in actuality, is equivalent to introducing into the law of compensation, derivatives of the error. But, in comparison with conventional differentiating devices, the described filter is more stable in the presence of interference.

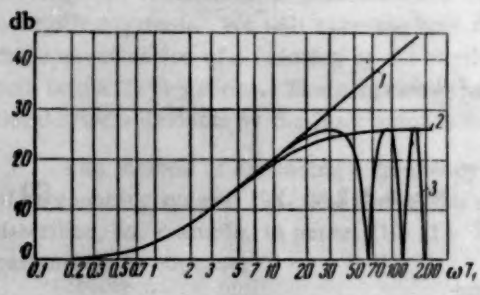


Fig. 3.

Thus, let us compare three differentiating devices from this point of view: an ideal differentiating circuit having the transfer function $K_1(p) = T_1p + 1$, a passive differentiating circuit with transfer function $K_2(p) = \frac{T_1p + 1}{0.05T_1p + 1}$ and a DEF with transfer function $D(p) = 10.5 - 9.5e^{-0.1T_1p}$. Over a certain range of frequencies the characteristics of all three devices practically coincide.

Figure 3 shows logarithmic plots of their amplitude characteristics.

It can be seen from Fig. 3 that in the case when noise is a component of the input signal, the DEF (curve 3) will ensure a better result than will the ideal differentiating circuit (curve 1) and the passive differentiating circuit (curve 2). If the noise spectrum is concentrated in the vicinity of the frequency ω_n, then the delay time of the delay element T should be chosen by means of the following relationship: $\omega_n T = 2k\pi$, where k is a positive whole number. If T is selected in this way the noise passes through the DEF without the increase which occurs in ordinary differentiating devices.

The DEF differs from other compensating networks in its flexibility with respect to changes in parameters. Calculated optimum parameters of a compensating unit do not always yield an optimum control process in a

practical system, since an idealized transfer function of the system is used in the calculations. Consequently, the filter parameters are made more precise by experimental means during alignment. This is very simple to achieve in a DEF by changing the settings of certain potentiometers.

Examples of Compensation of Automatic Control Systems

Example 1. Given a system, the circuit of which is shown in Fig. 4, it is required to determine a compensating DEF with optimum parameters.

We choose a first-order DEF with transfer function

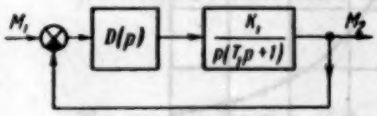


Fig. 4.

$$D(p) = a_0 + a_1 e^{-pT}.$$

The transfer function of the open-loop system will be

$$W(p) = \frac{K_1(a_0 + a_1 e^{-pT})}{p(pT_1 + 1)}.$$

Let

$$pT_1 = q,\ T = T_1\overline{T},\ K_1 a_0 T_1 = A_0,\ K_1 a_1 T_1 = A.$$

Substituting these relationships into the expression for W(p) we obtain

$$W(q) = \frac{A_0 + A_1 e^{-q\overline{T}}}{q(q+1)}.$$

The transfer function of the closed-loop system will be

$$K(q) = \frac{A_0 + A_1 e^{-q\overline{T}}}{q^2 + q + A_0 + A_1 e^{-q\overline{T}}} = \frac{A(q)}{B(q)}.$$

Let us find the derivatives of the numerator and denominator of the transfer function $K_c(q)$ when $q = 0$.

Derivatives of numerator	Derivatives of denominator	Derivatives of numerator	Derivatives of denominator
$\frac{d^0A}{dq^0} = A_0 + A_1$	$\frac{d^0B}{dq^0} = A_0 + A_1$	$\frac{d^3A}{dq^3} = -A_1\overline{T}^3$	$\frac{d^3B}{dq^3} = -A_1\overline{T}^3$
$\frac{dA}{dq} = -A_1\overline{T}$	$\frac{dB}{dq} = 1 - A_1\overline{T}$	$\frac{d^4A}{dq^4} = A_1\overline{T}^4$	$\frac{d^4B}{dq^4} = A_1\overline{T}^4$
$\frac{d^2A}{dq^2} = A_1\overline{T}^2$	$\frac{d^2B}{dq^2} = 2 + A_1\overline{T}^2$		

From relationships (6) and (7) for finding values of M_{2l} and Δ_{2l} and from the conditions for optima (11) we obtain

$$1 - 2A_0 - 2A_1(1 + \overline{T}) = 0 \text{ when } l = 2,$$

$$3 + A_1(\overline{T}^3 + 3\overline{T}^2) = 0 \quad \text{when } l = 4.$$

Solving this system of equations, we obtain

$$A_1 = -\frac{3}{\overline{T}^2(\overline{T}+3)}, \qquad A_0 = \frac{1}{2} + \frac{3(\overline{T}+1)}{\overline{T}^2(\overline{T}+3)}.$$

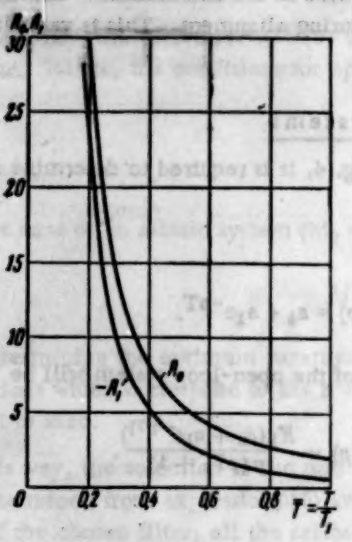


Fig. 5.

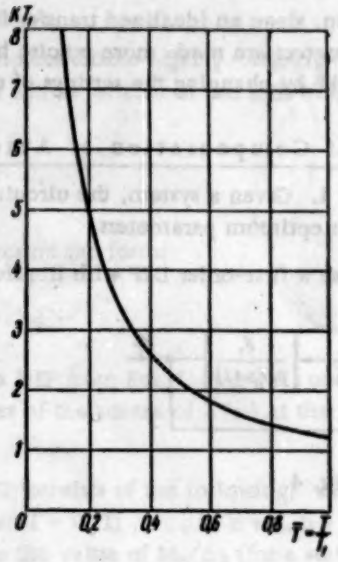


Fig. 6.

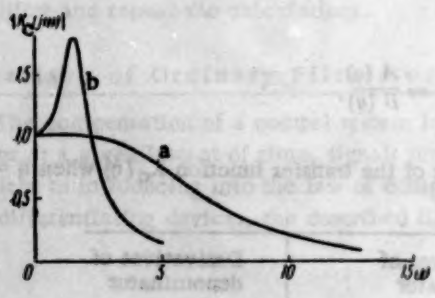


Fig. 7.

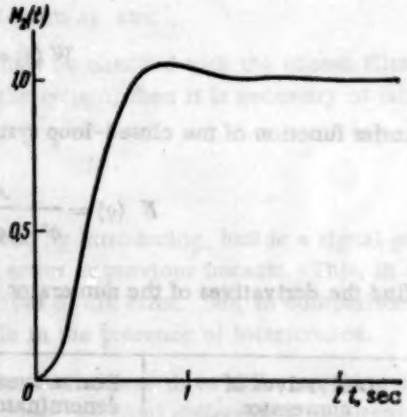


Fig. 8.

The amplification factor is

$$K = (a_0 + a_1)K_1 = 1/T_1(A_0 + A_1)$$

or

$$KT_1 = \frac{1}{2} + \frac{3}{\bar{T}(\bar{T}+3)}.$$

Variations of A_0, A_1 and KT_1 as functions of the relative delay time of the filter T are shown in Figs. 5 and 6.

Starting with the required value of the amplification factor K, we find the delay time of each element of the DEF from Fig. 6. The corresponding values of A_0 and A_1 of the filter can be determined from Fig. 5.

For example, if the controlled object has $K_1 = 1$ and $T_1 = 1$, while the required amplification factor $K = 3$, then from Figs. 5 and 6 we find $T = 0.36$, $a_1 = 9.9$, $a = -6.9$.

The transfer function of the chosen filter will be

$$D(p) = 9.9 - 6.9e^{-0.36p}.$$

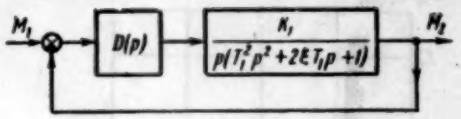


Fig. 9.

The amplitude-frequency characteristics of a compensated system (curve a) and an uncompensated system with the same amplification factor K = 3 (curve b) are shown in Fig. 7. The transient characteristic of the compensated system turns out to be slightly oscillatory (Fig. 8).

Example 2. Given a system, the circuit of which is shown in Fig. 9, it is required to determine a compensating DEF with optimum parameters.

We select a second-order filter with transfer function

$$D(p) = a_0 + a_1 e^{-pT} + a_2 e^{-2pT}.$$

The transfer function of the open-loop system will be

$$W(p=) \frac{K_1 (a_0 + a_1 e^{-pT} + a_2 e^{-2pT})}{p(T_1^2 p^2 + 2\xi T_1 p + 1)}.$$

Let $pT_1 = q$, $T = T_1\overline{T}$, $K_1T_1a_0 = A_0$, $K_1T_1a_1 = A_1$, $K_1T_1a_2 = A_2$.

Substituting these relationships into the expression for W(p), we obtain

$$W(q) = \frac{A_0 + A_1 e^{-q\overline{T}} + A_2 e^{-2q\overline{T}}}{q(q^2 + 2\xi q + 1)}.$$

The transfer function of the closed-loop system will be written in the form

$$K_C(q) = \frac{A_0 + A_1 e^{-q\overline{T}} + A_2 e^{-2q\overline{T}}}{q^3 + 2\xi q^2 + q + A_0 + A_1 e^{-q\overline{T}} + A_2 e^{-2q\overline{T}}} = \frac{A(q)}{B(q)}.$$

We find the derivatives of the numerator and denominator of the transfer function $K_C(q)$ when $q = 0$.

Derivatives of numerator	Derivatives of denominator
$\frac{d^0A}{dq^0} = A_0 + A_1 + A_2$	$\frac{d^0B}{dq^0} = A_0 + A_1 + A_2$
$\frac{dA}{dq} = -(A_1 + 2A_2)\overline{T}$	$\frac{dB}{dq} = 1 - (A_1 - 2A_2)\overline{T}$
$\frac{d^2A}{dq^2} = (A_1 + 4A_2)\overline{T}^2$	$\frac{d^2B}{dq^2} = 4\xi + (A_1 + 4A_2)\overline{T}^2$
$\frac{d^3A}{dq^3} = -(A_1 + 8A_2)\overline{T}^3$	$\frac{d^3B}{dq^3} = 6 - (A_1 + 8A_2)\overline{T}^3$
$\frac{d^4A}{dq^4} = (A_1 + 16A_2)\overline{T}^4$	$\frac{d^4B}{dq^4} = (A_1 + 16A_2)\overline{T}^4$
$\frac{d^5A}{dq^5} = -(A_1 + 32A_2)\overline{T}^5$	$\frac{d^5B}{dq^5} = -(A_1 + 32A_2)\overline{T}^5$
$\frac{d^6A}{dq^6} = (A_1 + 64A_2)\overline{T}^6$	$\frac{d^6B}{dq^6} = (A_1 + 64A_2)\overline{T}^6$

From relationships (6) and (7) for determining M_{2l} and Δ_{2l}, and from the conditions for optima (11) we obtain

$$4\xi A_0 + (4\xi + 2\overline{T})A_1 + (4\xi + 4\overline{T})A_2 = 1 \quad \text{for } l = 2,$$

$$(6\overline{T} + 6\xi\overline{T}^2 + \overline{T}^3)A_1 + (12T + 24\xi\overline{T}^2 + 8\overline{T}^3)A_2 = 6 - 12\xi^2 \quad \text{for } l = 4,$$

$$(20\overline{T}^3 + 10\xi\overline{T}^4 + \overline{T}^5)A_1 + (160\overline{T}^3 + 160\xi\overline{T}^4 + 32\overline{T}^5)A_2 = 60 \quad \text{for } l = 6.$$

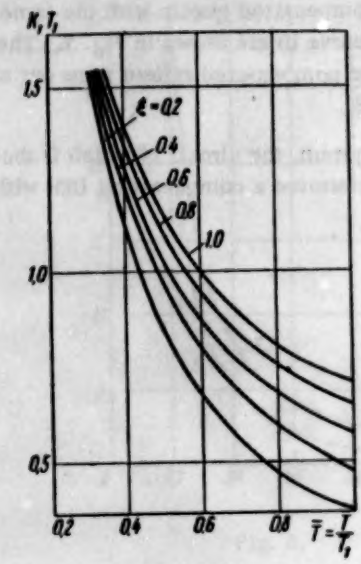


Fig. 10.

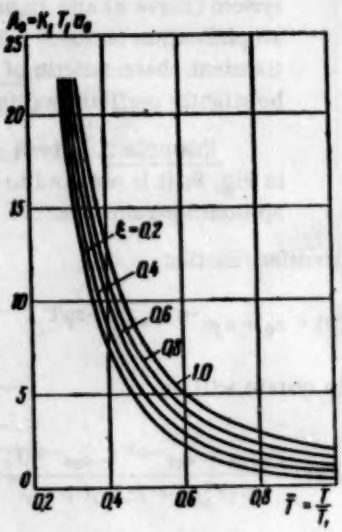
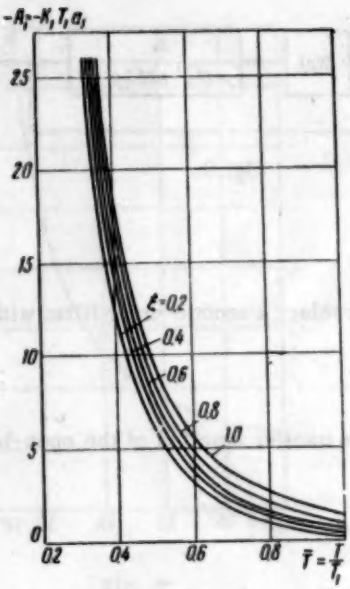


Fig. 11.



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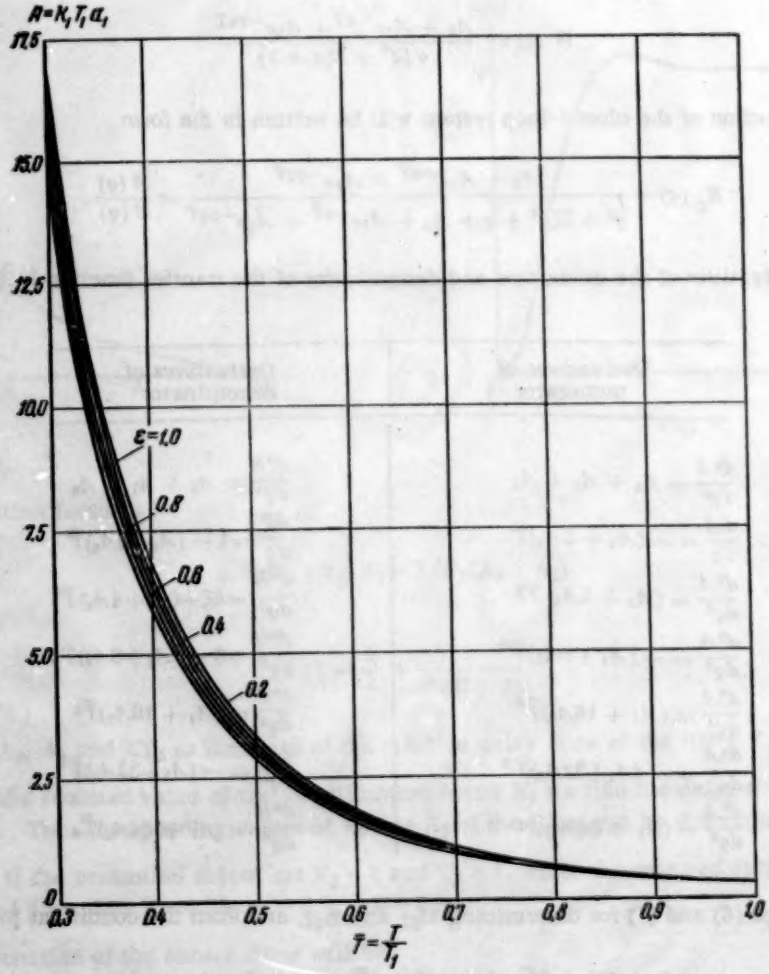

Fig. 13.

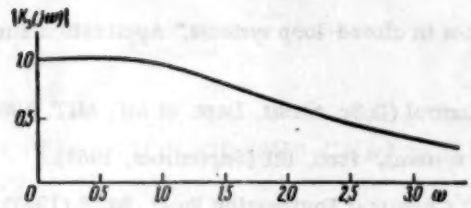

Fig. 14.

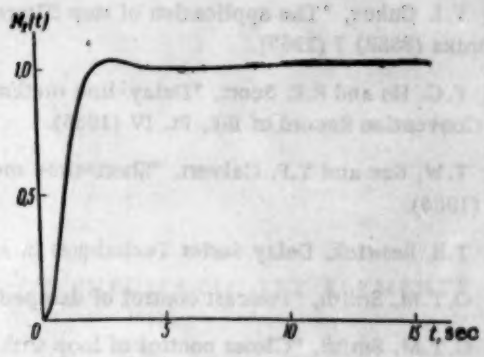

Fig. 15.

The amplification factor is

$$K(a_0 + a_1 + a_2)K_1 = 1/T_1(A_0 + A_1 + A_2)$$

or

$$KT_1 = A_0 + A_1 + A_2.$$

The variations of A_0, A_1, A_2, and KT_1 as functions of the relative delay time T of each element of the filter, for different values of ξ, are shown in Figs. 10-13. Starting with the required value of the amplification factor K one can, making use of these curves, determine all the filter parameters.

For example, if for the controlled object $K_1 = 1$, $T_1 = 1$, $\xi = 0.4$ and the required amplification factor $K = 1$, then making use of Figs. 10-13, we obtain $T = 0.5$, $A_0 = 4.71$, $a_1 = -6.90$, $a_2 = 3.17$.

The filter's transfer function will be

$$D(p) = 4.71 - 6.90e^{-0.5p} + 3.17e^{-p}.$$

The amplitude-frequency characteristic of the compensated system is shown in Fig. 14, while the transient characteristic is shown in Fig. 15. An uncompensated system with an amplification factor $K = 1$ is unstable.

SUMMARY

1. Delay element filters make it possible to improve the dynamic properties of automatic control systems.

2. Optimum filter parameters may be chosen by means of the method of approximation by an expansion of the frequency characteristic into a series (Braude's method).

Other restrictions imposed on the system, such as the specification of particular values of the error coefficients, can be taken into account when determining the filter parameters.

3. For the case of the compensation of simple closed-loop systems, including circuits with transfer functions

$$\frac{k_1}{p(pT_1 + 1)} \quad \text{and} \quad \frac{K_1}{p(T_1^2p^2 + 2\xi T_1 p + 1)},$$

formulas and curves have been obtained for determining filter parameters.

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INVESTIGATION OF THE CHARACTERISTICS OF PNEUMATIC JET ELEMENTS

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The article deals with an investigation of pneumatic jet elements, the operation of which is based on the utilization of the characteristics of pressure changes obtained when the impact tube is moved through different sections of the jet flowing from a nozzle. Examples are given of the possible applications of such elements in pneumatic devices.

Pneumatic devices are built at present, as a rule, with nozzle-vane type elements [1]. In the solution of many problems which arise in the field of pneumatic devices, the application of jet elements, henceforth referred to as nozzle-pipe elements, can be more effective. Their operation is based on the utilization of the characteristics of the distribution of pressure in an air jet.

Nozzle-pipe elements can be divided into two types. Elements in which the Pitot tube moves through the section of the jet emerging from the nozzle belong to the first type (Fig. 1a). To the second type belong elements in which there are no moving parts and variations in pressure on the pipe are brought about by the deflection of the air jet emerging from one of the nozzles, by a jet emerging from the other nozzle (Fig. 1b).*

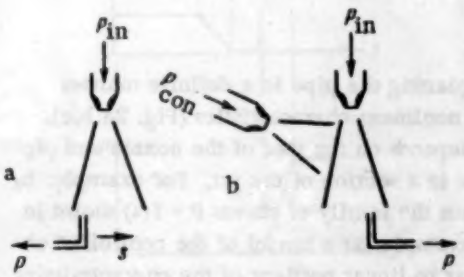


Fig. 1.

Jet elements have several advantages over elements of the nozzle-vane type in that there are no precise, ground details in them, it is not necessary to have hermetically sealed connections in them, and they work more reliably over a wide temperature range. A disadvantage of jet elements is their relatively high consumption of air. This last fact is insignificant if powerful sources of air exist in the controlled object itself (gas turbine and uniflow air-compressor motors, ergazlifts in the petroleum industry, etc.). In pneumatic equipment designed for general industrial application, the consumption of air can be reduced at the expense of a reduction in working pressures.

The use of nozzle-pipe elements of the first type is expedient in those cases when it is required to match variations in pressure with mechanical displacements. Nozzle-pipe elements of the second type are purely aerodynamic and serve to vary one pressure as a function of other pressures. In this paper nozzle-pipe elements of the first kind are investigated. In part 1 examples are given of their possible application in pneumatic devices. The following parts are devoted to the theory of these elements.

The characteristics of variations in pressure occurring on the receiving tube as it moves through different sections of the air jet emerging from the nozzle are investigated. Specific problems arise in this section, in connection with the small dimensions of nozzles in nozzle-pipe elements, and in connection with the possible effect

*Concerning the use of jet elements in pneumatic devices see also [2].

of the tube dimensions on the obtained characteristics. Section 2 clarifies the extent to which one can, in calculating the characteristics of nozzle-pipe elements, make use of data related to free jets. The analysis of functional transformations carried out by elements of the type under consideration is given in part 3. In section 4, possible divergences between the characteristics of nozzle-pipe elements and the characteristics of free jets are examined further. Here consideration is given to questions having special significance in cases where the relative dimensions of the receiving end of the pipe are so large (in connection with the small dimensions of the nozzle opening) that they cannot be considered negligibly small in comparison with the dimensions of the jet's cross section.

1. Examples of the Possible Use of Nozzle-Pipe Type Elements in Pneumatic Devices

Nozzle-pipe elements of the type being examined can be used effectively in devices for the transformation of mechanical displacements into air-pressure variations and in devices for the reverse transformation.

In the transformation from mechanical displacements into changes in air pressure, the problem of obtaining linear characteristics is sometimes set; more often, however, the corresponding device must achieve functions of a nonlinear converter.

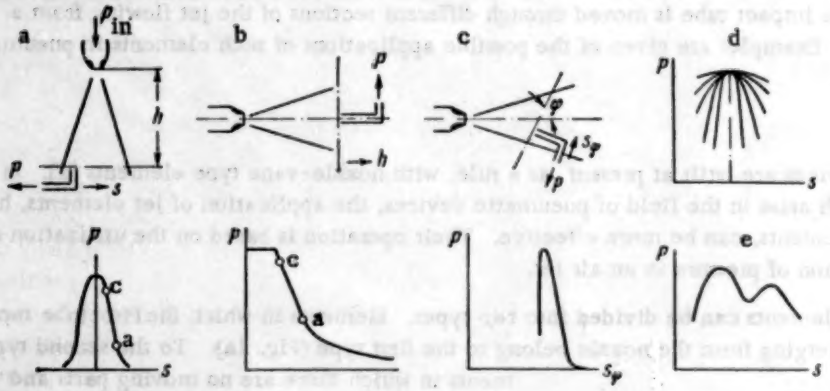


Fig. 2.

The simplest converter is a single nozzle-pipe element. By displacing the pipe in a definite manner through different sections of the jet one can obtain different shapes of nonlinear characteristics (Fig. 2a,b,c). The shape of the characteristic P = f(s) or the characteristic P = f(h) depends on the size of the nozzle and pipe, on the input (supply) pressure, and on the initial placement of the pipe in a section of the jet. For example, by varying the input pressure and the placement distance h, one can obtain the family of curves P = f(s) shown in Fig. 2d. An element which yields such characteristics can serve, for example, as a model of the controlled object in the simulation of optimalizing control processes.* By using near to linear portions of the characteristics indicated above (see, for example, regions ab in Figs. 2a and b) one can carry out transformations which are practically linear. Using only one nozzle-pipe element one can carry out functional transformations for the achievement of which it is often necessary to use a complex to produce and finish cam mechanisms, lever mechanisms, etc.

Nonlinear characteristics of a more complex form, for example, characteristics P = f(s) with two or more extremums (Fig. 2e) or others, can be reproduced by the combination of two or several elements of the type indicated earlier on.

Nozzle-pipe elements can also be used to build a universal, nonlinear converter reproducing any given

*In the simulation, the displacement of the maximum of the characteristic P = f(s) along the s axis can be produced by changing the position of the supply nozzle, and along the P axis by changing the supply pressure P_{in}. The static characteristics of the object are considered here. However, when a pneumatic chamber or a system of pneumatic chambers is connected to the output of the given element, the dynamic properties of the object can be reproduced on the model as well.

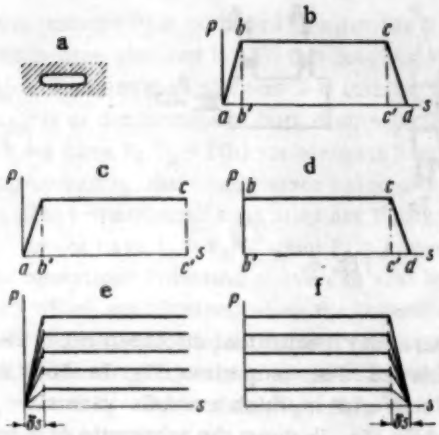


Fig. 3.

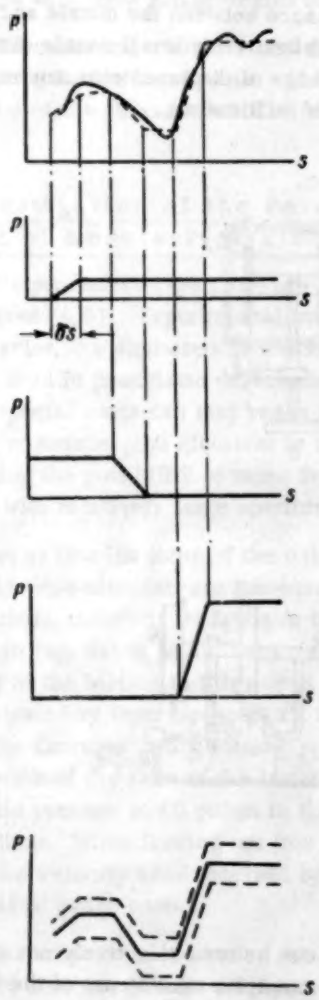


Fig. 4.

characteristic P = f(s). A characteristic P = f(s) of an arbitrary form can be obtained by the use of a set of nozzle-pipe elements with nozzles having an elongated cross section (Fig. 3a).

For each of these nozzles the characteristic P = f(s) has a shape like the one shown in Fig. 3b for displacements of the pipe along the major axis of the nozzle's cross section. If the movement of the pipe is restricted to the regions ab'c' or b'c'd the resultant characteristics are those shown in Figs. 3c and d. As the input pressure is varied the level of the horizontal section changes and, accordingly, the inclination of the side branches of the characteristics indicated earlier (Figs. 3e and f) change.

Let us examine the method of obtaining the characteristic P = f(s) having an arbitrary form, by means of a set of elements with nozzles of the above-mentioned form. Let, for example, the curve shown in the upper part of Fig. 4 be the specified one. Let us replace this curve by a piecewise-linear approximation. This we will construct in such a way that the projection of each piece of the approximation onto the s axis be equal to δs (the value of δs is determined from the characteristics shown in Fig. 3e and f). The three middle graphs of Fig. 4 show elementary characteristics, each of which has a sloping region equal in length and angle of inclination to the corresponding section of the piecewise-linear approximation described above. Each of these elementary characteristics is obtained by means of one of the nozzles. Their required relative position along the s axis, shown in Fig. 4, is obtained by coordinating the position of the nozzles during their initial installation (Fig. 5) while the required inclination of the sloping region of each of the elementary characteristics is obtained by setting a particular air pressure in the supply for the corresponding nozzle. The pressures received by the pipes of separate elements are added by means of a pneumatic, interthrottle chamber which fulfills the function of a summator. The pressure at the output of the summator changes in accordance with a characteristic which corresponds to the given piecewise-linearized characteristic P = f(s) with, possibly, the only difference being that the whole characteristic can be displaced by a constant amount of pressure, to one or the other side of the specified one (see the characteristics shown by dotted lines on the lower graph of Fig. 4). The characteristic can be brought to the specified level either by the use of a nozzle-vane (Fig. 6a) or by means of an arrangement containing no moving parts and based on purely aerodynamic action (Fig. 6b). In the first case the difference between the pressure supplied to the given element and the pressure at its output is created by the tightening of springs by a regulating screw. In the second case the functions of a displacement element is carried out by a pneumatic chamber analogous to the one used as a summator. As the pressure P_k varies, the quantity P_{out} follows the variations of P_k displaced by a constant value of pressure, determined by the adjustment pressure P_{adj}. When the given adjustment element is connected to the summator, shown in Fig. 5, one obtains the characteristic P_{out} = = f(s) displaced relative to the characteristic P_k = f(s) by a constant amount depending on P_{adj}.

When jet elements of the type being considered are used instead of nozzle-vane elements, the transforma-

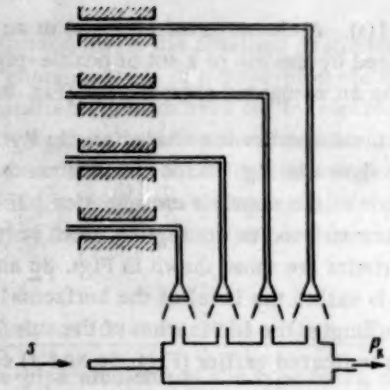


Fig. 5.

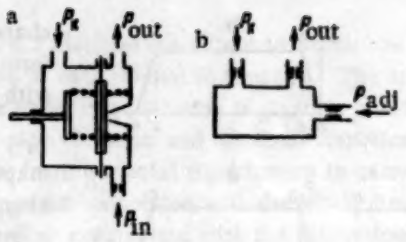


Fig. 6.

tion of air pressure into mechanical displacement is also more easily achieved. For comparison, Fig. 7a shows the schematic of a converter in which a nozzle-vane element is used, while Fig. 7b shows the schematic of a converter using a nozzle-pipe element. In the latter case, nozzle-pipe 1 is used as a feedback element. As a result of the operation of the feedback loop there exists a strict correspondence between the value of the pressure at the input to chamber 2 (this same pressure is produced, as a result of the feedback loop, in chamber 3) and the variations in the distance between the nozzle and pipe. By locating the pipe in different sections of the jet one can design, even with relatively small nozzle dimensions, conversion programs for a relatively large mechanical displacement. The range of displacements can vary from tenths of a millimeter to several millimeters or to values measured in tens of millimeters.

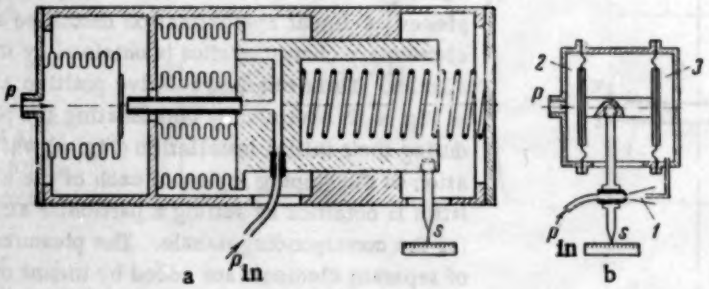


Fig. 7.

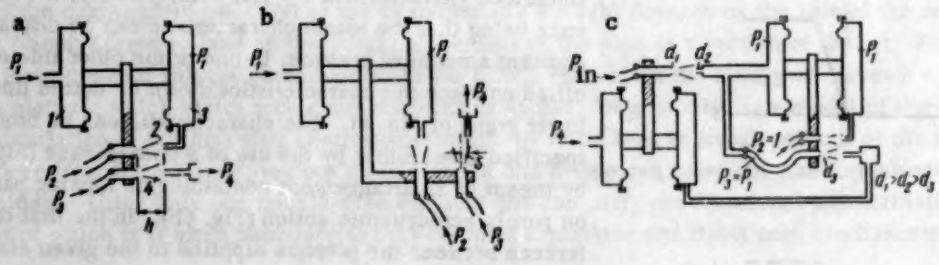


Fig. 8.

A study of the characteristics of jets shows that nozzle-pipe elements can be used effectively not only to carry out the above-mentioned functions, but also for other objectives. For example, making use of the fact that for every given position of the pipe the ratio of the pressure on the pipe to the pressure in front of the nozzle does not depend on the value of the latter (we have in mind excess pressures), one can use nozzle-pipe elements to carry out multiplication, division and squaring operations. Figure 8a shows a unit which illustrates the principle of the execution of these operations with nozzle-pipe elements.* When pressure P_1 is applied to chamber 1, the

*Compare this with the schematic of the computer presented in [3], which is used to carry out those same functions and which is made up of nozzle-vane elements.

same pressure P_1 is produced in chamber 3 as a result of the operation of the feedback system which includes nozzle-pipe element 2. To this feedback system P_2 is the supply pressure. On the same movable crosspiece on which the nozzle of element 2 is installed there is also the nozzle of another nozzle-pipe element 4. The diameters of the nozzles of both elements are the same as are the dimensions of the pipes. For every given value of $\underline{h}$ we have $P_2/P_1 = f(h)$ for element 2 and $P_3/P_4 = f(h)$ is maintained for element 4. Since, with the feedback loop operating, the steady-state value of $\underline{h}$ is the same for both elements, the values of f(h) in the right sides of the above-mentioned equalities are the same. Hence, $P_2/P_1 = P_3/P_4$, whence it follows that $P_4 = P_1P_3/P_2$. When $P_2 = 1$* we have $P_4 = P_1P_3$, when $P_3 = 1$, correspondingly, $P_4 = P_1/P_2$. However, if $P_2 = 1$ and $P_1 = P_3$, then $P_4 = P_1^2$. The operations indicated above can also be carried out if one makes use of the characteristics of pressure variations which are obtained when the receiving end of the pipe is moved along the jet's cross section (Fig. 8b). If an extra feedback loop is introduced, the system shown in Fig. 8a can be converted in such a way that the operation of the extraction of a square root can be carried out. For example, for the circuit shown in Fig. 8c, where $P_2 = 1$ and $P_3 = P_1$, we have $P_4 = P_1P_3/P_2 = P_1^2$ or $P_1 = \sqrt{P_4}$, where P_4 is the pressure at the input and P_1 is the pressure at the output.

Models of the universal, nonlinear converter and multiply-divide unit described above were built, using nozzle-pipe elements,** at the laboratory of pneumohydraulic automatic control of the IAT AN SSSR. Experimentation with these arrangements confirmed the correctness of the design of their principal systems.

The examined examples of the possible use of nozzle-pipe elements in pneumatic instruments can be extended to hydraulic instruments. As it is shown below, the theory and design of nozzle-pipe elements are based on the use of the characteristics of free, turbulent jets, but these characteristics are common to air and drop liquids.

2. Investigation of the Possibility of Using the Characteristics of Free Jets in the Design of Nozzle-Pipe Elements

The processes of the propagation of a free jet have been the subject of special investigations in aerodynamics (see [4, 5]). Experimental investigations in these studies were basically carried out on the flow of air from nozzles, the diameters of which were measured in units of the order of several centimeters or decimeters. Nozzles used in pneumatic devices can have diameters not exceeding tenths of millimeters in size, and only in certain special cases can they reach sizes of the order of several millimeters. In the investigation of the characteristics of nozzle-pipe elements in the role of primary elements in pneumatic devices there arises the question concerning the possibility of using the experience accumulated from the investigation of the flow of air from nozzles with relatively large aperture-cross sections.

Let us first list some of the data concerning the free jets with the characteristics of which the characteristics of nozzle-pipe elements are subsequently compared. The profile of the longitudinal section of a free, axially symmetrical, turbulent jet is shown in Fig. 9a. The initial region, of length h_H, contains a core of constant speeds (shaded in Fig. 9a) in which the axial speed of air particles is equal to their speed at the exit from the nozzle. The rest of the section in this region is occupied by a boundary layer. When $h > h_H$ in the main region of the jet, the boundary layer occupies all the cross section of the jet and the jet's axial speed decreases with increase in $\underline{h}$. The decrease in the particle speed occurs for a constant amount of motion in the jet, and is dependent on the increase of the mass of the latter as a result of the drawing of air into the jet from the surrounding medium. The static pressure at all points in the free jet is assumed to be constant and equal to the pressure of the surrounding medium. When flowing out into the atmosphere, this is the atmospheric pressure and, in that case, the pressure of the velocity head received by the pipe introduced into the jet appears together with the whole pressure measured in gauge units.

The scheme for producing jets that is described above is a simplified one. In actual fact, the static pressure in the jet does not remain strictly constant. In the main region the intensity of the turbulent flow changes and, in accordance with this, one should divide the main region into separate zones. The characteristics of the jet

*The value of the pressure equal to unity can be taken to be any pressure; experiments were carried out on multiply-divide units built from the described schematic with $P_2 = 1$ atmos.

**In the experimental model the unit containing the movable parts was mounted on a ribbon suspension to ensure displacement of the nozzles without skewing; the force was transmitted, as is shown in Fig. 8, by membranes.

depend on the distribution of velocities in the output section of the nozzle. However, the effect of the above-mentioned factors is usually relatively small, and for free, turbulent jets, one can make use of certain average data on the basis of which one can calculate the characteristics of such jets.

Additional processing of the experimental characteristics listed in earlier works dealing with the theory of free jets was carried out to clarify the numerical values of quantities determining the jet's profile. The data obtained in this way was compared with the data of experiments carried out with nozzle-pipe elements in the course of the present investigation. The following considerations were kept in mind in the course of this.

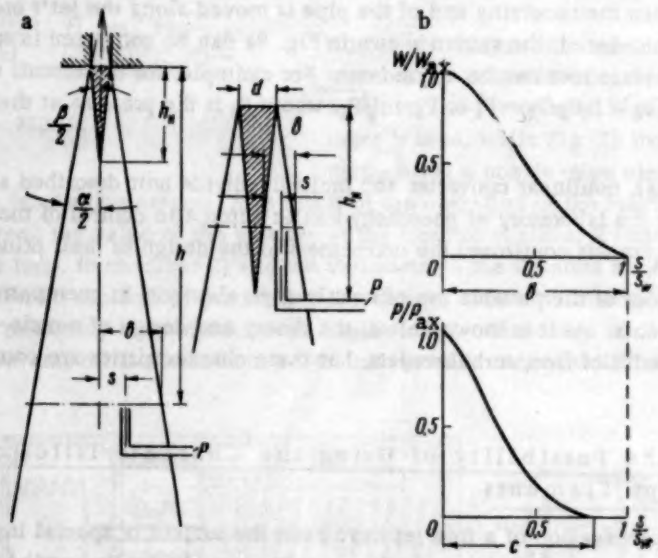


Fig. 9.

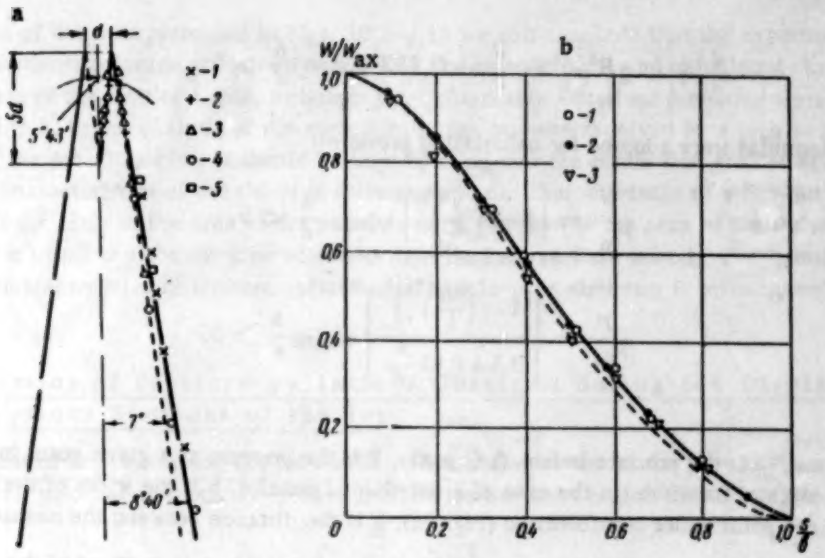
1. In order to clarify the operating conditions of nozzle-pipe elements it is interesting to investigate the distribution of pressures in the jet. Works connected with the investigation of free jets, however, usually present characteristics of velocity distribution which, however, are also plotted on the basis of pressure measurements. Since the total pressure at any point in a jet, measured in gauge units, appears together with that velocity pressure, it is proportional to the square of the speed at a given point. Consequently, the practical boundaries of the jet, determined, for example, from the condition of equality on the boundary of the speed or pressure 0.5% from their maximum values, are different in those cases when velocity curves and pressure curves are being examined (compare, respectively, the quantities b and c in Fig. 9b; in Fig. 9b s_W is the value of s for which W is equal to 0.5% of W_{ax} in the given section of the jet).

2. In conducting experiments with free jets the axis of the tube is usually placed parallel to the axis of the jet. Hence, although the velocity vectors in the jet are not collinear, in making use of formulas for the calculation of the distribution of velocities, derived on the basis of experimental characteristics of pressure distribution, one can assume that at every point in the jet the axial component of velocity is equal to the total velocity.

Let us first indicate the mean, calculated data with which the experimental characteristics of free jets and nozzle-pipe elements were compared. This data was obtained on the basis of fundamental relationships of the theory of turbulent, free jets, and the preliminary processing of experimental data on free jets.* The following numerical values were taken for the quantities determining the boundaries of the jet (see Fig. 9a): $\frac{1}{2}\alpha = 8°40'$ (for the speed curve) or $\frac{1}{2}\alpha = 7°$ (for the pressure curve); $h_H = 5d$ which corresponds to $\frac{1}{2}\beta = 5°43'$. In conjunction with this it was assumed that the width of the boundary layer, determined as shown in Fig. 9a, is equal to

$$b = h\,\mathrm{tg}\,5°43' + h\,\mathrm{tg}\,8°40' = 0{,}252\,h \quad \text{for} \quad \frac{h}{d} < 5, \tag{1}$$

*Data on free jets obtained by Tryupel', Fertman, and from experiments at the Goettingen laboratory of aerodynamics was processed further; for references to the respective sources see [4].


a
d
5d
5°43'
7°
8°40'
×-1
+-2
△-3
○-4
□-5
b
W/W ax
1,0
0,8
0,6
0,4
0,2
0
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0,4
0,6
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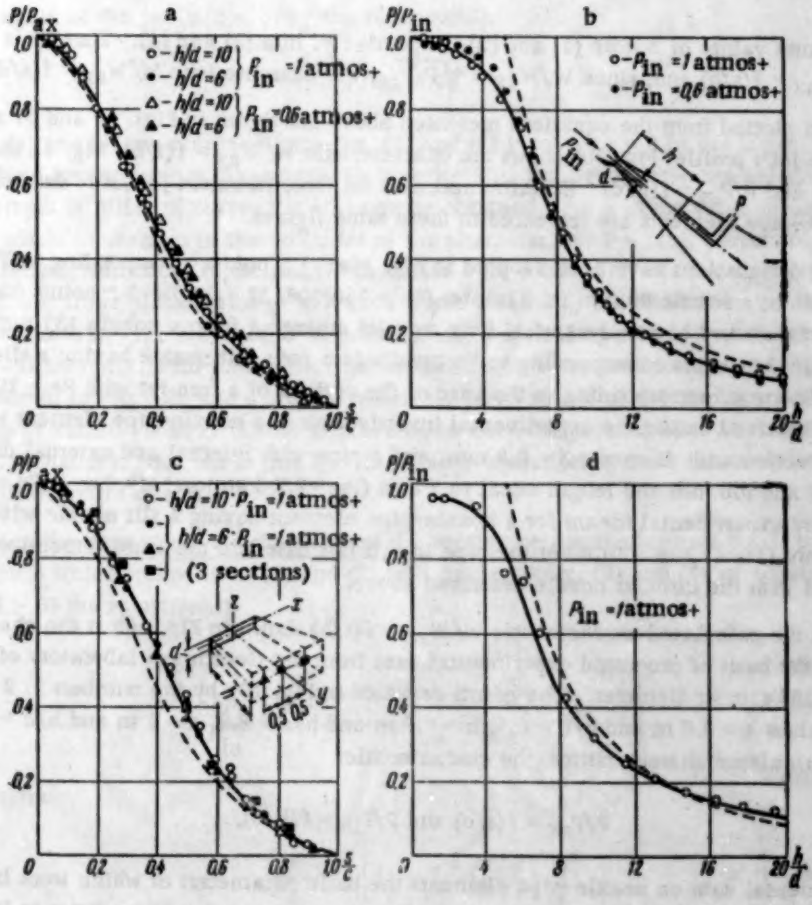
Fig. 10.


a
P/P ax
○- h/d=10 } P in =1 atmos+
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b
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d
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P
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(3 sections)
z
x
y
0,5
0,5
d
d
P in =1 atmos+
0
0,2
0,4
0,6
0,8
10 r/R
4
8
12
16
20 h/d

Fig. 11.

$$b = \frac{d}{2} + h \operatorname{tg} 8°40' = \frac{d}{2} + 0{,}152h \quad \text{for} \quad \frac{h}{d} > 5. \tag{2}$$

The following formulas were adopted for calculating pressures:

$$\frac{P}{P_{in}} = \left[1 - \left(\frac{s}{b}\right)^{3/2}\right]^4 \text{ where } \frac{h}{d} < 5, \tag{3}$$

$$\frac{P}{P_{in}} = \left\{\frac{\left[1 - \left(\frac{s}{b}\right)^{3/2}\right]^2}{0.3 + 0.14\frac{h}{d}}\right\}^2 \text{ where } \frac{h}{d} > 5. \tag{4}$$

In these equations P_{in} is the pressure before the nozzle, P is the pressure at a given point in the jet, d is the diameter of the nozzle or its width (in the case of a slit-shaped nozzle), b is the width of the boundary layer, s is the coordinate of the point under consideration (Fig. 9a), h is the distance between the nozzle and pipe.

Keeping in mind the characteristics of pressure and velocity distributions in sections of the jet normal to its axis, which are examined below we introduce the designations W_{ax} and P_{ax} corresponding, respectively, to the velocity and pressure at the center of the jet's cross section, i.e., at the point in the section lying on the jet's axis.

We can substitute values of b from (1) and (2), respectively, into (3) and (4). Thus, it is not hard to obtain the relationship $P/P_{ax} = f(s/b)$ and, since $W/W_{ax} = \sqrt{P/P_{ax}}$, the characteristic $W/W_{ax} = f(s/d)$.

Characteristics plotted from the equations presented above are shown in Figs. 10 and 11 as dotted lines. Figure 10a shows the jet's profile, Fig. 10b shows the characteristic $W/W_{ax} = f(s/b)$, Fig. 11 shows the characteristics $P/P_{ax} = f(s/c)$ and $P/P_{in} = f(h/d)$. Experimental data for free, turbulent jets, and data obtained in the investigation of nozzle-pipe elements are presented in these same figures.

The following designations have been adopted in Fig. 10a: 1 — points corresponding to Tryupel's data on a free jet emerging from a nozzle 90 mm in diameter (Re = 550,000, M = 0.26); 2 — points corresponding to data from the Goettingen laboratory of aerodynamics, for a free jet emerging from a nozzle 137 mm in diameter (Re = = 380,000, M = 0.12); 3 — points corresponding to Fertman's data for a slit nozzle having a slit width of 30 mm and a slit length of 650 mm, corresponding to the case of the outflow of a free jet with Re = 70,000 and M = = 0.09; 4 — points obtained during the experimental investigation of a nozzle-pipe element with a nozzle having a circular cross section with diameter d = 0.8 mm, and a pipe with internal and external diameters, respectively, equal to 0.27 and 0.5 mm and length equal to 7 mm (P_{in} = 0.2-1 atmos+; M= 0.4-1, Re = 7,500-31,000); 5 — points obtained by experimental means for a nozzle-pipe element having a slit nozzle with slit width 0.31 mm and slit length 1.2 mm (M ≈ 1, Re = 10,000); the pipe in this last case had the same dimensions as the one in the nozzle-pipe element with the circular nozzle described above.

Together with the calculated characteristic $W/W_{ax} = f(s/b)$ shown in Fig. 10b is the characteristic $W/W_{ax} = f(s/b)$, plotted on the basis of processed experimental data from the Goettingen laboratory of aerodynamics obtained for a nozzle 137 mm in diameter. The points designed in Fig. 10b by the numbers 1, 2 and 3 relate, respectively, to the values h = 0.6 m and h/d = 6.7, h = 0.8 m and h/d = 8.9, h = 1 m and h/d = 11.1.** Figure 11 shows, besides the calculated characteristics, the characteristics

$$P/P_{ax} = f(s/c) \text{ and } P/P_{in} = f(h/d),$$

plotted from experimental data on nozzle-pipe elements the basic parameters of which were indicated above. The characteristics in Figs. 11a and b are plotted for a circular nozzle, the characteristics in Fig. 11c and d were plotted for a nozzle having an exit aperture in the shape of a slit. In Fig. 11c are plotted points, for h/d = 6, obtained from the pipe's crossing of the jet at three different transverse sections, shown in the nozzle's sketch.

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*Pressure in excess of atmospheric pressure.

**For sections of a jet at a large distance from the nozzle, the characteristic $W/W_{ax} = (f)(s/b)$, for low outflow velocities, approaches a straight line in the region between s/b = 0.3 and s/b = 1.

On the basis of the data presented in Figs. 10 and 11 we can conclude that the experimental characteristics of nozzle-pipe elements coincide sufficiently well with the experimental and calculated characteristics corresponding to the case of the flow of a free, turbulent jet. The design equations presented above are used subsequently in section 3, in the analysis of the variations in the pressure received by a pipe as it is moved in different sections of the jet. However, it should be kept in mind that the conclusion arrived at concerning the coincidence of the characteristics of nozzle-pipe elements and the characteristics of a free jet is valid only under the condition that the ratio of the area of the nozzle's output section to the area of the jet's section does not exceed the values for which the nozzle-pipe elements described above were tested. The question of the effect of this ratio on the characteristics of pressure variations in nozzle-pipe elements is investigated separately in section 4.

3. Characteristics of Pressure Variations Obtained during the Displacement of the Pipe Along Various Sections of the Jet

Displacement of the pipe along the jet's axis. When the apertures of the pipe and a nozzle having a round section are coaxial, the characteristic $P = f(h)$ is defined for the main region of the jet ($h/d < 5$) by the relationship

$$\frac{P}{P_{in}} = \frac{1}{\left(0.3 + 0.14\,\frac{h}{d}\right)^2}, \tag{5}$$

and for the initial region of the jet ($h/d < 5$) by the relationship

$$P/P_{in} = 1. \tag{6}$$

Relationships (5) and (6) are obtained from Eqs. (3) and (4) if we set $s = 0$ in the latter. The characteristic $P/P_{in} = f(h/d)$, defined by expressions (5) and (6), is shown in Fig. 12a. From this characteristic, plotted in relative coordinates, various families of curves $P = f(h)$ can be obtained. For a given value of the nozzle diameter d, variations in P_{in} result in changes in the ordinates of the characteristic $P = f(h)$, proportional to the value of P_{in} (Fig. 12b).* However, variations in the nozzle diameter d for a constant value of P_{in} is equivalent to variations in the scale of quantities plotted along the h axis: for a given P_{in}, the curves obtained for nozzles of different diameters are shown in Fig. 12d. A row of other families of characteristics $P = f(h)$ can be obtained if P_{in} and d are varied simultaneously, with their variations connected by some functional relationship. A feature of the characteristics shown in Fig. 12b and d are the relatively large values of the displacements h, corresponding to the working range of variations in P. These displacements have magnitudes of several millimeters or even tens of millimeters. What is significant is that the linear approximation of these characteristics is possible over sections several millimeters in length.

Displacement of the pipe along the diameter of the section perpendicular to the axis of the jet. Substituting the expression for b which follow from (1) and (2) into, respectively, (3) and (4) we obtain for the main region of the jet ($h/d > 5$) the relationship

$$\frac{P}{P_{in}} = \left\{\frac{\left[1 - 2.83\left(\frac{s/d}{1 + 0.304\,h/d}\right)^{3/2}\right]^2}{0.3 + 0.14\,h/d}\right\}^2 \tag{7}$$

and for its initial region

$$\frac{P}{P_{in}} = \left[1 - 7.9\left(\frac{s/d}{h/d}\right)^{3/2}\right]^4. \tag{8}$$

For each given value of h/d the relationship $P/P_{in} = f(s/d)$ is determined from (7) or (8). Figure 12e shows,

*Taking the direction in which the tube approaches the nozzle to be the positive direction for changes in h, and choosing in a corresponding manner the origin of the h scale, we will obtain the characteristics shown in Fig. 12c.

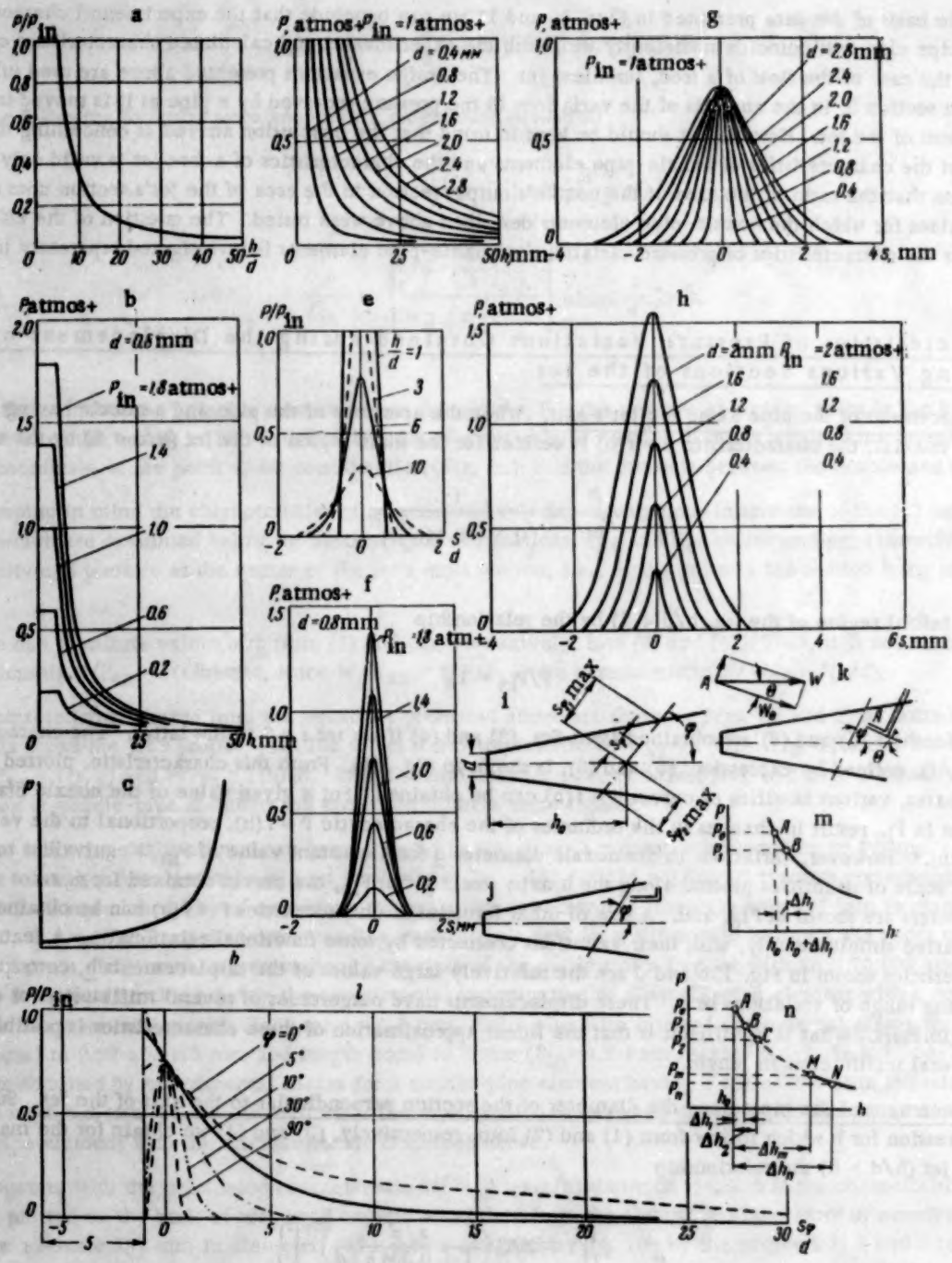

Fig. 12.

as examples, the characteristic $P/P_{in} = f(s/d)$ plotted for h/d = 1, 3, 6, and 10. Even by using any one of these characteristics a series of families of characteristics $P = f(s)$ can be obtained. For example, for a nozzle with d = 0.8 mm and h/d = 6; i.e., with h = 4.8 mm, we will obtain, taking different values of P_{in}, a family of curves $P = f(s)$ shown in Fig. 12f. Keeping P_{in} = 1 atmos+ we obtain, for different d and for a given value of h/d, the family of curves shown in Fig. 12g. For proportional changes in the quantities d and P_{in} we obtain from the original characteristic $P/P_{in} = f(s/d)$ with h/d = 6, shown in Fig. 12e, the family of curves $P = f(s)$ shown in Fig. 12h, etc.

Movement of the pipe in a plane forming an angle φ with the jet's axis. We will limit our investigation

to the case where the pipe moves along the major axis of the ellipse obtained when the main region of the jet is cut by a plane forming an angle φ with the jet's axis (Fig. 121). Moreover

$$\frac{P}{P_{in}} = \left\{ \frac{\left[1 - 2.83\left(\frac{(s_\varphi / d)\sin\varphi}{1 + 0.304\,(h_0 / d \pm (s_\varphi / d)\cos\varphi)}\right)^{5/3}\right]^2}{0.3 + 0.14\,(h_0 / d \pm (s_\varphi / d)\cos\varphi)} \right\}^2. \tag{9}$$

The values of the quantities φ, s_φ, h_0 are given in Fig. 121; the rest of the quantities in (9) are the same as those appearing in earlier expressions. Figure 121 also shows the limiting values of s_φ, being, respectively, $s_{\varphi_1}\max$ and $s_{\varphi_2}\max$. When $0 < s_\varphi < s_{\varphi_1}\max$ the sign in front of the term $(s_\varphi/d)\cos\varphi$ in (9) should be taken as plus, while when $0 < s_\varphi < s_{\varphi_2}\max$, the sign should be taken as minus. The quantities $s_{\varphi_1}\max$ and $s_{\varphi_2}\max$ can be determined from a drawing of the jet's profile (Fig. 10a) or they can be calculated from the equations

$$\frac{s_{\varphi_1}\max}{d} = \frac{0.152\,\frac{h_0}{d} + 0.5}{\cos\varphi\,(\operatorname{tg}\varphi - 0.152)} \quad \text{and} \quad \frac{s_{\varphi_2}\max}{d} = \frac{0.152\,\frac{h_0}{d} + 0.5}{\cos\varphi\,(\operatorname{tg}\varphi + 0.152)}. \tag{10}$$

The last expression in (10) is valid only in those cases when the plane of the jet's section does not touch the jet's initial region, which corresponds to the condition $\tan\varphi \geq \frac{1.26}{(h_0/d) - 5}$. When $\tan\varphi < \frac{1.26}{(h_0/d) - 5}$ one should use the equation

$$\frac{s_{\varphi_2}\max}{d} = \frac{\frac{h_0}{d} - 5}{\cos\varphi}. \tag{11}$$

The derivation of expressions (9), (10) and (11) is given in Part 1 of the Appendix. We note that when $\varphi = \pi/2$, we obtain Eq. (7) as a particular case of (9); here $h_0 = h$ and $s_\varphi = s$. When $\varphi = 0$, we obtain formula (5) as another particular case of (9); here $h_0 \pm s_\varphi = h$.

Figure 121 shows the characteristics $P/P_{in} = f(s_\varphi/d)$, obtained from Eq. (9) for $h_0/d = 6$ when $\varphi = 0$, 5, 10, 30, and 90°.

Further expansion of the class of characteristics $P = f(s)$ obtained in sections of an axially symmetrical, turbulent jet. Families of characteristics $P = f(s)$ different from the ones considered above can be obtained if the pipe is displaced along the chords rather than the diameter of a section perpendicular to the jet's axis. In this case, the characteristic $P_{in}/P = f(s/d)$ is determined, for a given h/d, from the formula

$$\frac{P}{P_{in}} = \left\{ \frac{\left[1 - 2.83\left(\frac{\sqrt{(s_x / d)^2 + (a / d)^2}}{1 + 0{,}304\,h / d}\right)^{5/3}\right]^2}{0.3 + 0.14\,h / d} \right\}^2, \tag{12}$$

in which s_x is the distance from the section's axis, measured along the chord, and a is the distance of the given chord from the jet's axis. For $s_x\max/d$ we obtain, in this case, the equation

$$\frac{s_x\max}{d} = \sqrt{\left(\frac{b}{d}\right)^2 - \left(\frac{a}{d}\right)^2}. \tag{13}$$

The pipe can be displaced along an axis parallel to the jet's axis and separated from it by a distance s. The family of characteristics obtained in this case is described by Eq. (7); one must only take in this equation the value of s/d as constant and consider h/d as the variable quantity. The minimum value of the ratio h/d for which the pipe, moving in the direction of the nozzle, reaches the boundary of the jet, is equal to

$$\frac{h_{min}}{d} = \frac{\frac{s}{d} - 0.5}{0.152}. \tag{14}$$

The derivation of Eqs. (12)-(14), as well as further comments on the use of Eq. (7) in the case under consideration, are given in Part 2 of the Appendix. The class of families of characteristics $P = f(s)$ can be widened even further if the pipe is displaced along the chords of curves formed by the intersection of inclined planes with the jet.

All the relationships presented above refer to the case where the pipe's axis is parallel to the jet's axis (Fig. 12j) where, as it was mentioned, it is assumed that the total velocity pressure of the jet acts on the pipe. The pipe can be situated at an angle θ to the jet's axis (Fig. 12k). However, for large angles of inclination of the pipe, the disturbances introduced into the air flow by the pipe become appreciable, and in performing calculations it is necessary to introduce an additional correction to the value of the velocity head, determined from the speed $W_\theta = W_{\cos\theta}$. In such a case, for example, (7) is transformed into the expression

$$\frac{P}{P_{in}} = k\left\{\frac{\left[1-2.83\left(\frac{s/d}{1+0.304\,h/d}\right)^{3/2}\right]^2}{0.3+0.14\,h/d}\cos\theta\right\}^2, \tag{15}$$

where k is a correction coefficient. On the basis of experiments conducted with a pipe having internal and external diameters equal to, respectively, 0.27 and 0.5 mm (nozzle diameter 0.8 mm) this coefficient can be determined from the expression*

$$k = 1-(\theta/70)^{2.34}. \tag{16}$$

In this way, another variable quantity θ is introduced into the expression for the functional dependence of P/P_{in} on s/d and h/d determined previously from Eq. (7). In a particular case, when $\theta = 90 - \varphi$, in (15) $\cos\theta$ is replaced by $\sin\varphi$.

Expressions (5), (7), (9), and all subsequent ones were obtained for the main region of an axially symmetrical jet. Analogous expressions can be derived also for the initial region of the jet.

Calculations of the parameters of a nozzle-pipe element. We will limit ourselves to examining the characteristics obtained when the receiving aperture of the pipe is moved along the jet's axis, and determined by Eq. (5).** First let us present formulas for the calculation of the distance from the nozzle to the pipe h_0 and the pressure before the nozzle P_{in}, for which the curve $P = f(h)$ passes through the given points $A(P_0, h_0)$ and $B(P_1, h_0 + \Delta h_1)$ (Fig. 12m)***:

$$h_0 = \frac{\Delta h_1}{\sqrt{\frac{P_0}{P_1}-1}} - 2.14\,d, \tag{17}$$

$$P_{in} = \frac{P_0}{\left(\sqrt{\frac{P_0}{P_1}-1}\right)^2\left(\frac{0.14\,\Delta h_1}{d}\right)^2} \tag{18}$$

Let us illustrate the use of Eqs. (17) and (18) by the following numerical examples. Let $P_0 = 0.65$ atmos+, $P_1 = 0.38$ atmos+, $\Delta h_1 = 2$ mm. Let us take $d = 0.8$ mm. From Eqs. (17) and (18) we find $h_0 \approx 4.7$ mm and $P_{in} = 0.83$ atmos+.

We can specify that the characteristic $P = f(h)$ passes through the points $A(P_0, h_0)$, $B(P_1, h_0 + \Delta h_1)$, $C(P_2, h_0 + \Delta h_2)$, . . . , $M(P_m, h_0 + \Delta h_m)$, $N(P_n, h_0 + \Delta h_n)$, the coordinates of which satisfy the condition (19).

We note further that if it is more convenient, from a practical point of view, to specify the characteristic $P = f(h)$ by the angular coefficient $(dP/dh)_0$ of the tangent to the characteristic at the point $P = P_0$, then the quantities h_0 and P_{in} can be calculated from Eqs. (20) and (21).

*For more detail see next section.

**Only in this case does the calculation of the nozzle-pipe element's parameters become sufficiently simple. In other cases the calculation reduces to the solution of nonlinear equations, which can be carried out only by graphical means.

***The derivation of formulas (17), (18), and formulas (19)-(21) which are given further, is given in Part 3 of the Appendix.

$$\frac{\Delta h_1}{\Delta h_2} = \frac{\sqrt{\frac{P_0}{P_1}} - 1}{\sqrt{\frac{P_0}{P_2}} - 1}, \ldots, \frac{\Delta h_m}{\Delta h_n} = \frac{\sqrt{\frac{P_0}{P_m}} - 1}{\sqrt{\frac{P_0}{P_{in}}} - 1}. \quad (19)$$

$$h_0 = \frac{2P_0}{\left|\left(\frac{dP}{dh}\right)_0\right|} - 2.14\,d \quad (20)$$

and

$$P_{in} = \frac{0.28\,P_0^2}{\left|\left(\frac{dP}{dh}\right)_0\right|}. \quad (21)$$

Comments about the characteristics of slit-shaped nozzles. In part 2 it was shown that the characteristics of slit nozzles (Fig. 3a) obtained when the receiving aperture of the pipe is moved along the jet's axis, or those obtained when the pipe moves in a transverse section of the jet along the straight line parallel to the z axis (Fig. 11c) are also closely approximated by the design equations presented earlier. In these cases d, in the given equations, is the width of the slit. For practical applications it may be important to determine the characteristics obtained when the pipe's receiving end is moved along the major axis of the jet's transverse cross section (along the axis of the jet's transverse cross section, lying in the xy plane, see Fig. 11c). By the use of just such

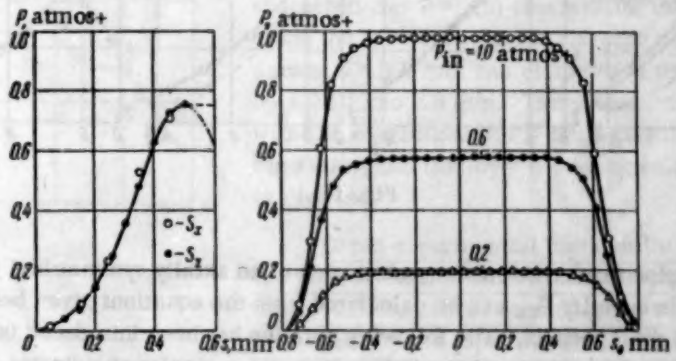

Fig. 13.

characteristics, the universal nonlinear converter, using nozzle-pipe elements, described in part 1 was designed. Experimental investigations showed that the distribution of pressures in the boundary layer of the jet flowing out from the slit nozzle (we have in mind the inclined portions of the characteristics), differs little for the cases where the pipe moves along the transverse cross section's central axes parallel to the z axis and the x axis. Figure 13a shows, as an example, the corresponding regions of the characteristics $P = f(s_z)$ and $P = f(s_x)$ plotted for an element with a nozzle having a cross section 0.31 mm wide and 1.2 mm long, when h = 1.8 mm and P_{in} = =1 atmos+. Figure 13b shows the experimental characteristics $P = f(s_x)$ for the same element with h = 0.3 mm and P_{in} = 1, 0.6, and 0.2 atmos+. These characteristics are very close in their shape to the theoretical characteristics shown earlier in Fig. 3. The experiments were carried out with a pipe having an internal cross section diameter of 0.27 mm and an external diameter of 0.5 mm.

4. Investigation of the Deviations of Nozzle-Pipe Element Characteristics from the Characteristics of Free, Turbulent Jets

Deviations of the characteristics of nozzle-pipe elements from the characteristics of a free jet can be brought about by the following factors. With relatively large dimensions of the pipe's receiving aperture (with small nozzle dimensions) the presence of gradients in the velocity head pressures in the jet's section assume significance. In this case the pressure measured by the pipe is some mean of the pressures acting on the plane of its

receiving aperture. Moreover, when a tube of relatively large dimensions is introduced into the jet, the variation in the pattern of flow in the jet in front of the pipe, which is brought about by the action of aerodynamic factors, can have an effect on the pressure measured by the pipe.

Investigation of the effect of the pipe's receiving aperture. First we will assume that in the region in front of the pipe's receiving aperture the velocity field is the same as it would be in a free jet, i.e., that the pipe does not produce disturbances in the jet, and we will assume that the process by which the pipe slows down the air is isentropic. (Later, possible deviations from this idealized case will be discussed.) Under the above assumptions the energy of the slowed-down flow is characterized by the quantity $= \int_f P df$, $P_{ve} f$, where P_{ve} is the pressure measured by the pipe, P is the pressure at some point in the receiving aperture.*

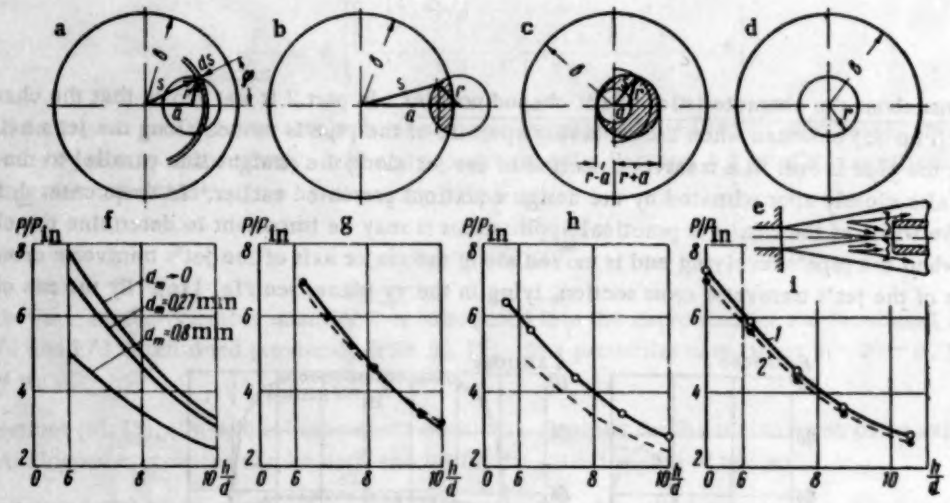

Fig. 14.

For positions of the pipe in sections of the main region of an axially symmetrical jet shown, respectively, in Fig. 14a, b, c, and d, the quantity P_{ve} can be calculated from the equations given below, the derivation of which is given in Part 4 of the Appendix. The following notation has been introduced in Fig. 14: b – radius of the jet's section, a – distance from pipe's axis to jet's axis, s and φ – polar coordinates of points in the jet, which lie in the field of the pipe's receiving aperture. Also, in the formulas given below there appear, as before, d – nozzle diameter, h – distance between the ends of the nozzle and pipe, P_{in} – air pressure supplied to the nozzle.

For the case given in Fig. 14a:

$$P_{ve} = \frac{2d^2 P_{in}}{\pi r^2} \int_{\frac{a-r}{d}}^{\frac{a+r}{d}} \left\{ \frac{\left[1-\left(\frac{2\ s/d}{1+0.304\ h/d}\right)^{3/2}\right]^2}{0.3+0.14\ h/d} \right\}^2 \frac{s}{d} \arccos \frac{a^2+s^2-r^2}{2\ as}\, d\left(\frac{s}{d}\right). \qquad (22)$$

Equation (22) also holds for the case shown in Fig. 14b with the sole difference that the upper and lower limits of integration are replaced, respectively, by b/d and a/d – r/d. For the location of the pipe in the jet, shown in Fig. 14c:

$$P_{ve} = \frac{2d^2 P_{in}}{r^2} \int_{0}^{\frac{r-a}{d}} \left\{ \frac{\left[1-\left(\frac{2\ s/d}{1+0.304\ h/d}\right)^{3/2}\right]^2}{0.3+0.14\ h/d} \right\}^2 \frac{s}{d}\, d\left(\frac{s}{d}\right) +$$

*We will note once more that P_{ve} and P are the pressures of the velocity head and are, at the same time, the total pressures measured in excess units.

$$+\int_{\frac{a-r}{d}}^{\frac{a+r}{d}}\left\{\frac{\left[1-\left(\frac{2\,s/d}{1+0.304\,h/d}\right)^{5/3}\right]^2}{0.3+0.14\,h/d}\right\}^2\frac{s}{d}\arccos\frac{a^2+s^2-r^2}{2as}\,d\left(\frac{s}{d}\right). \quad (23)$$

Equation (23) can also be used for the case shown in Fig. 14d if the upper limit in the first integral on its right side is taken as r/d and the second integral is set to zero.

For the location of the pipe in the jet shown in Fig. 14a, calculations from Eq. (22) are carried out for given values of h/d and r/d in the following way. The limits of variations in a/d are taken into account (in the case being considered r/d < a/d < (b − r)/d and a few values of a/d which lie within these limits are selected. The limits of integration are calculated for every chosen value of a/d, and several values of s/d which fit between these limits are taken. The subintegral function is calculated for them. Its graph is plotted point-by-point, and a planimeter is used to determine the value of the integral in Eq. (22). These calculations and plots are repeated for other chosen values of a/d. The considerations presented above also apply to the calculation of the second integral on the right side of (23). The subintegral function of the first integral on the right side of (23) does not contain a/d and the calculation of this integral reduces to merely picking several values of s/d between the limits 0 < s/d < (r/d − a/d), plotting the curve of the subintegral function, and determining the area bounded by the curve by a planimeter.

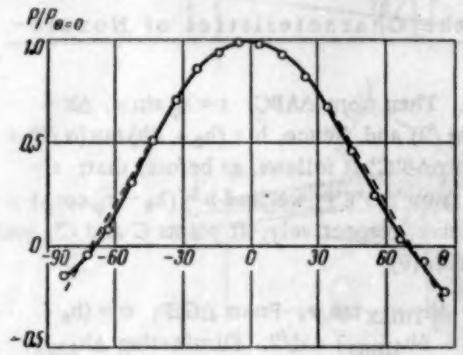

Fig. 15.

The method described above was used to calculate the characteristics P = f(h) obtained for displacements of the pipe along the jet's axis, for nozzle-pipe elements with nozzle diameter d = 0.8 mm and diameter of the pipe's internal section d_p = 0.27 and 0.8 mm. These characteristics were compared with the characteristic P = f(h), calculated from Eq. (5) under the assumption that d_p = 0. All three characteristics are shown in Fig. 14f.

Given experimental methods for determining the effect of the pipe dimension on the characteristics of a nozzle-pipe element. Considerations regarding the effect of the disturbances introduced by the pipe into the jet flow. For a nozzle-pipe element with a pipe having d_p = 0.27 mm, Fig. 14g shows the experimentally determined (solid line) characteristic P/P_{in} = f(h/d) and the characteristic P/P_{in} = f(h/d) calculated by the method described above (dotted curve). These characteristics are practically the same. Figure 14h shows the experimental (solid) and calculated (dotted) characteristics P/P_{in} = f(h/d) for a nozzle-pipe element with d_p = 0.8 mm (as before, the nozzle diameter d = = 0.8 mm). This experimental characteristic passes lower than the experimental curve for the pipe with d_p = = 0.27 mm (Fig. 14g). Thus, the tendency for further decrease in the value of P with increasing d_p is confirmed, but the observed values of P are higher than the calculated ones. The latter can be explained by the fact that beside the effect of the area of the pipe's aperture cross section on the flow characteristics of nozzle-pipe elements, which was discussed above, the effect of disturbances introduced by the pipe into the stream can, as has already been mentioned, begin to appear. When the losses during deceleration are taken into account, the pressure received by the pipe must be smaller than that obtained from calculations carried out for the case of isentropic deceleration. However, an increase in pressure can occur as a result of the deformation of the streamlines in the jet (as a result of which the energy of a greater part of the jet than that which was taken into account in the description of the calculating system, is transformed into the velocity head pressure. An increase in pressure can also occur as a result of the introduction into the deceleration region under consideration, of particles from layers adjacent to it (Fig. 14e). In this case, it is probable that not only the internal but also the external pipe diameter affects the characteristics of the nozzle-pipe elements.* For the pipe with d_p = 0.27 mm, the outside diameter

*The pipe's length and the profile of its leading end can also affect its streamline conditions. All experiments were conducted with pipes of length l = 7 mm, and having their end surfaces perpendicular to the pipe axis.

was equal to 0.5 mm for the experiments described above, while for the pipe with d_p = 0.8 mm the outside diameter was 1.26 mm. The area corresponding to the outside diameter for the first of these pipes was 2.5 times smaller, and for the second 2.5 times larger than the cross-sectional area of the nozzle. Experiments were carried out with a pipe having an external diameter equal to 3 mm and d_p = 0.8 mm to determine what effect further increases in the pipe's external diameter have. The obtained characteristics (1 – for a pipe with a 3 mm external diameter, 2 – for a pipe with a 1.26 mm external diameter) are shown in Fig. 14i; they practically do not differ from each other.

All the experimental characteristics described earlier were obtained with the receiving aperture of the pipe situated in a plane perpendicular to the jet's axis. As it was already noted, if the pipe's angular position is varied it is necessary to take into account in the calculations the variation of the pipe's streamline conditions as a function of the angle θ (Fig. 12k). Figure 15 shows the experimental characteristic $P/P_{\theta=0} = f(\theta)$, obtained by rotating the pipe about a vertical axis passing through the center of its receiving aperture, with this point situated on the jet's axis. Experiments were carried out with a nozzle having d = 0.8 mm, and a pipe with internal diameter 0.27 mm and external diameter 0.5 mm, with h/d = 6. The dotted line in Fig. 15 shows the corresponding calculated curve obtained by calculating $\underline{k}$ from Eq. (16).

APPENDIX

Derivation of Formulas Used in the Calculation of the Characteristics of Nozzle-Pipe Elements

1. Derivation of expressions (9)-(11). Let $h = h_0 + \Delta h$ (Fig. 16a). Then from ΔABC: $s = s_\varphi \sin\varphi$, $\Delta h = s_\varphi \cos\varphi$ and $h = h_0 + s_\varphi \cos\varphi$. From ΔDEF: $b - d/2 = (h_0 + \Delta h)\tan(\alpha/2)$ and, hence, $b = (h_0 + \Delta h)\tan(\alpha/2) + d/2 = (h_0 + s_\varphi \cos\varphi)\,0.152 + d/2$. If $h = h_0 - \Delta h$ (Fig. 16b) then from $\Delta AB'C'$ it follows, as before, that: $s = s_\varphi \sin\varphi$ and $\Delta h = s_\varphi \cos\varphi$. However, in this case $h = h_0 - s_\varphi \cos\varphi$. From $\Delta D'E'F'$ we find $b = (h_0 - s_\varphi \cos\varphi) \times (0.152) + \frac{1}{2}d$. Making use of Eq. (7) for determining the pressure, respectively, at points C and C', and substituting into (7) the values of $\underline{s}$, $\underline{h}$, and $\underline{b}$ shown above we end up with (9).

Expressions (10) are obtained thus. From ΔAGH (Fig. 16c): $b_1 = \Delta h_{1\max}\tan\varphi$. From ΔGEF: $c = (h_0 + \Delta h_{1\max})\tan(\alpha/2) = (h_0 + \Delta h_{1\max})\,0.152$. In this case $b_1 = 0.152(h_0 + \Delta h_{1\max}) + d/2$. Eliminating $\Delta h_{1\max}$ from the two expressions for b_1 we find $b_1 = (0.152h_0 + d/2)\tan\varphi/(\tan\varphi - 0.152)$. From ΔAGH: $s_{\varphi_{1\max}} = b_1/\sin\varphi$.

Substituting the value of b_1 into the last expression we obtain the first equation in (10). An equation for determining $s_{\varphi_{2\max}}/d$ is derived in an analogous manner. However, the last is valid only up to the limiting position of the section plane shown in Fig. 16d, where this plane intersects the jet's boundary at the point with abscissa 5d, which corresponds to the transition from the initial region to the main region of the jet. For this position we find from ΔOKL (Fig. 16d) $f = 5d\tan\alpha/2 = 0.76d$. In this case NL = 1.26d and from ΔALN it follows that $\tan\varphi = 1.26/(h_0/d - 5)$. When $\tan\varphi < 1.26/(h_0/d - 5)$, which follows from Fig. 16e, $s_{\varphi_{2\max}} = (h_0 - 5d)/\cos\varphi$ and $s_{\varphi_{2\max}}/d = (h_0/d - 5)/\cos\varphi$, i.e., we obtain expression (11).

2. Directions for the derivation of Eqs. (12)-(14) and the use of Eq. (7) in the case of the displacement of the pipe along the axis parallel to the jet's axis. Equation (12) is obtained from (7) by replacing $\underline{s}$ by $\sqrt{s^2x + a^2}$ (Fig. 16f). Expression (13) follows from the relationship $s_{x\max} = \sqrt{b^2 - a^2}$ (Fig. 16g). An examination of Fig. 16h leads us to conclude that it is possible to make use of Eq. (7) in the case of the displacement of the pipe along an axis displaced by a distance $\underline{s}$ from the jet's axis. From ΔEFD we find $(s - \frac{1}{2}d)/h_{\min} = \tan\alpha/2 = 0.152$, whence (14) follows.

3. Derivation of Eqs. (17)-(21). First we will indicate the method of deriving Eqs. (17) and (18). From (5) $P_0 = P_{in}\dfrac{1}{(0.3 + 0.14h_0/d)^2}$ and $P_1 = P_{in}\dfrac{1}{\left(0.3 + 0.14\dfrac{h_0 + \Delta h_1}{2}\right)^2}$. Hence, $P_0/P_1 = \left(\dfrac{0.3 + 0.14[h_0 + \Delta h_1]/d}{0.3 + 0.14h_0/d}\right)^2$. Solving the last equation with respect to h_0, we arrive at (17). Substituting the obtained value of h_0 into the expression $P_0 = P_{in}\dfrac{1}{(0.3 + 0.14h_0/d)^2}$, we obtain (18). Let us further examine the characteristic shown in Fig. 12n. Analogously to the way in which Eq. (17) was derived we can find the expression $h_0 = \Delta h_2/(\sqrt{P_0/P_2} - 1) - 2.14d$. This last equation and Eq. (17) are common only under the condition that $\Delta h_1/(\sqrt{P_0/P_1} - 1) = \Delta h_2/(\sqrt{P_0/P_1} - 1)$. Equations for each section up to $h_0 + \Delta h_n$ can be treated analogously. As a result we will obtain conditions (19). Equation (20) is derived as follows: we differentiate (5) from the expressions

$$\left(\frac{dP}{dh}\right)_0 = -\frac{0.28\,P_{in}}{(0.3+0.14\,h_0/d)^2\,d} \quad \text{and} \quad P_0 = P_{in}\,\frac{1}{(0.3+0.14\,h_0/d)^2};$$

we eliminate P_{in} and, taking into account that $(dP/dh)_0 < 0$ and, therefore, $|(dP/dh)_0| = -(dP/dh)_0$, we arrive at (20). Equation (21) is obtained by substituting the value of h_0 from (20) into the expression $P_0 = P_{in}\,\frac{1}{(0.3+0.14h_0/d)^2}$.

4. Derivation of Eqs. (22) and (23). When the tube is situated in a section of the jet, as shown in Fig. 14a, the pressure is the same at all points in the shaded area [it is determined from Eq. (7)], but the area of this region is equal to $f = 2s\varphi\,ds$. Because $\cos\varphi = (a^2 + s^2 - r^2)/2as$, then $f = 2s\cos([a^2 + s^2 - r^2]/2as)\,ds$.

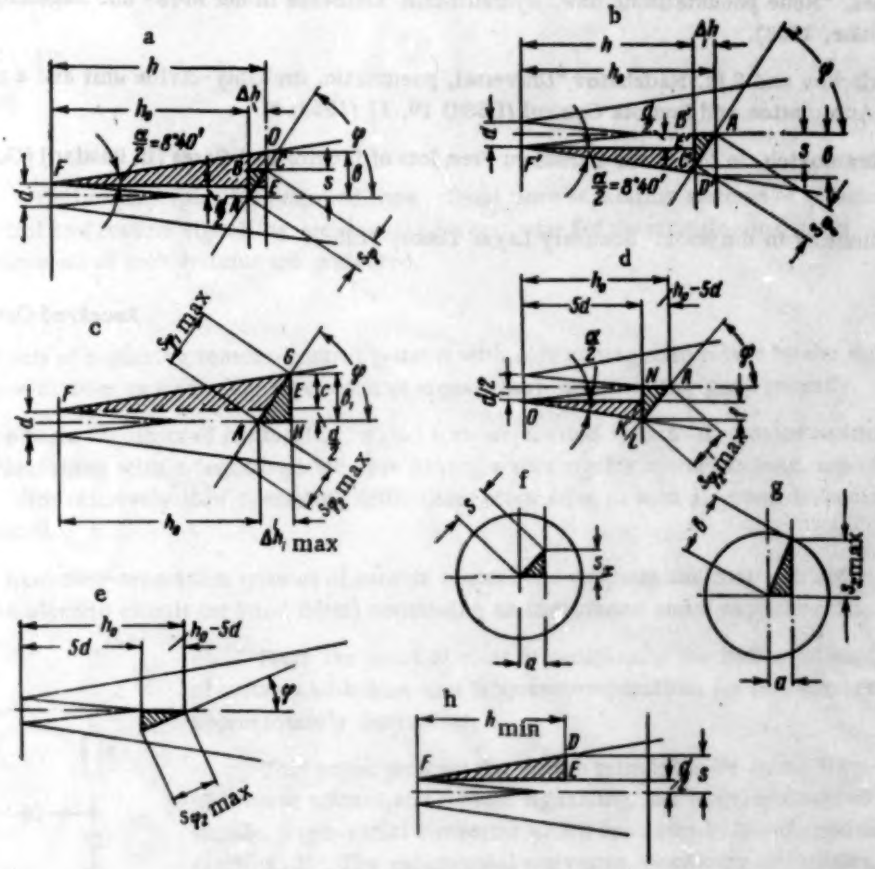

Fig. 16.

An elementary force acting on this area is equal to

$$P_{in}\left\{\frac{\left[1-\left(\frac{2s/d}{1+0{,}304\,h/d}\right)^{1/3}\right]^2}{0{,}3+0{,}14\,h/d}\right\}^2 2s\,\arccos\left(\frac{a^2+s^2-r^2}{2as}\right)ds.$$

Replacing s, a and r by the ratios s/d, a/d and r/d, integrating with respect to s/d between the limits of $a/d - r/d$ and $a/d + r/d$, and dividing the resultant force thus obtained by the area of the pipe's aperture πr^2, we obtain (22). It follows from Fig. 14b that the derivation presented above includes the given case, $s/d = a/d - r/d$ and $s/d = b/d$ are only the limits of integration.

For the case given in Fig. 14c, one takes the sum of two integrals, one of which corresponds to the interval $0 < s < r < a$, and the other to the interval $r - a < s < r + a$ (the shaded region in Fig. 14c corresponds to the last interval). The derivation of Eq. (23) is analogous to the derivation of Eq. (22). The second integral in Eq. (23)

is the same as the integral in Eq. (22), and it corresponds to the second of the intervals indicated above. The elementary areas for the first interval are determined by the expression $f = 2\pi s\,ds$, which is used in determining the first integral in the right side of (23) instead of the expression $f = 2s \arccos([a^2 + s^2 - r^2]/2as)\,ds$, which was given before. The coaxial location of the apertures of the pipe and nozzle (Fig. 14d) can be considered to be a particular case of the relative location of the pipe in a section of the jet shown in Fig. 14c. In this case, only the first of the integrals in Eq. (23) remains and the upper limit of integration is equal to r/d, since a = 0.

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DESIGN PRINCIPLES FOR CONTACTLESS REMOTE-CONTROL SYSTEMS WITH EXPONENTIAL CONVERTERS*

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Design principles for proposed, new, signal time-separation systems of remote control and remote signalling are described. Formulas for determining the main parameters of such systems are presented.

The prospects of replacing remote-control systems with electromagnetic relays by the more reliable contactless systems with time- or frequency-separation of signals have become very good recently.

One of the basic elements of contactless, signal time-separation systems of remote control consists of a multiwinding transformer with a ferromagnetic core having a rectangular hysteresis loop, crystal diodes, and capacitors [1]. With relatively slow operation, difficulties often arise in such a system in matching it to a communication channel.

In signal frequency-separation systems of remote control the simplest and most promising frequency discriminator is the electric circuit (or band filter) containing an inductance and a capacitor [2].

From the point of view of complexity the indicated contactless elements of systems with time- and frequency-separation, for the same requirements, are approximately equivalent.

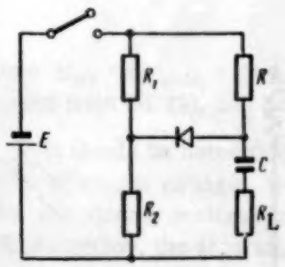

Fig. 1.

This paper presents the design principles for signal time-separation systems of remote control and remote signalling, the basic element of which is a new, simple, exponential converter which has already found application in telemetry systems [3]. The exponential converter, consisting of resistors, a capacitor, and a diode, permits one to design, in a number of cases, simpler systems with time- and code-separation of signals, without the use of inductances or transformers. Such systems can be well matched to a communication channel, they are very stable and can be made smaller and lighter. This makes their prospects of development very promising.

The simplest circuit of an exponential converter which transforms the magnitude of a resistance or capacitance into a pulse duration is shown in Fig. 1. The converter consists of a bridge, one arm of which contains resistors R_1 and R_2, while the other contains a resistance R and a capacitor C. A diode is connected across a diagonal of the bridge. The forward resistance of the diode R_F and the resistance R are chosen from the conditions $R_F \ll R$ and $R_2 \ll R$. The diode's reverse resistance R_{re} must be considerably higher than the resistance R.

When rectangular pulses (Fig. 2a) derived from a blocking-oscillator or some other pulse generator, are applied to the bridge the voltage across capacitor C changes as shown in Fig. 2b while the current through the capacitor changes as shown in Fig. 2c. At time t_c the voltage across the capacitor becomes equal to the voltage

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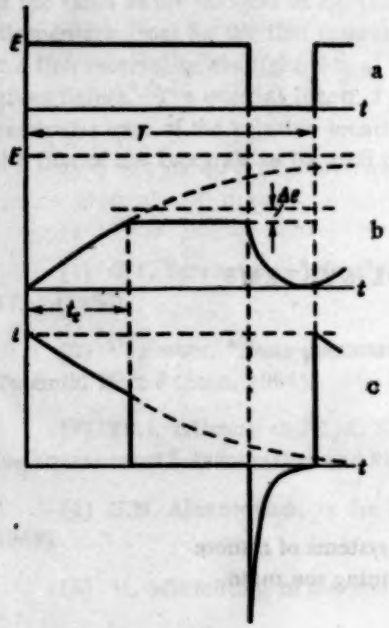

Fig. 2.

across resistor R_2, the diode begins to conduct and, for the chosen parameters and $R_L \ll R$, the capacitor stops charging. The duration of the converter's output pulses is determined by the expression

$$t_c = RC \ln \frac{R_1 + R_2}{R_1}. \qquad (1)$$

The trailing edge of the pulses lengthens if the diode does not have an ideal piecewise linear characteristic. In practical converters durations of the trailing edge of the order of 1% of the maximum duration of the output pulses can be obtained. The duration of the trailing edge will be

$$\Delta t = \frac{\Delta E}{E} RC \frac{R_1 + R_2}{R_1}, \qquad (2)$$

where E is the voltage amplitude of the primary pulses, ΔE are approximate voltage variations along the diode's characteristic, for which the diode's internal resistance changes from R/2 to 2R.

In the simplest system of remote control which makes use of a distributing code with n commands, each command is given a different pulse duration by the exponential converter. The object switch K in the transmitter (Fig. 3, left) switches resistors and gives the pulses a particular duration t_{ci} (resistor R or capacitor C can be varied). The converter's output voltage is delivered from the load resistor $R_L \ll R$ to the linear repeating relay LR, which can be a transistorized, electronic relay, etc.

The right side of Fig. 3 shows the circuit of a three-command receiver. The receiver consists of a distributor in the form of a complex, exponential converter with four identical RC networks, a differentiating stage DS and a separate coincidence circuit CC for each command.

The time plots of voltages across load resistors R_{L1}, R_{L2}, R_{L3} and R_{L4} are shown in Fig. 4b, c, d and e, while the output pulses from the distributor, which are delivered to the inputs of the coincidence circuits, are shown in Fig. 4f, g and h.

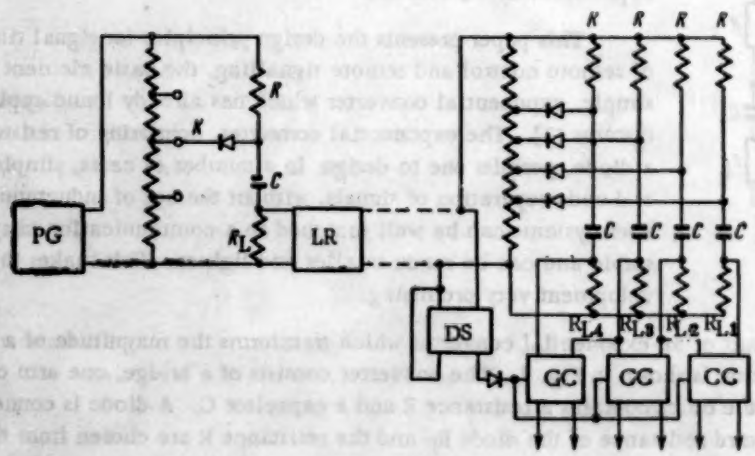

Fig. 3.

A pulse corresponding to the trailing edge of the pulse transmitted from the transmitter is delivered from the output of the differentiating stage to all the second inputs of the coincidence circuits. If the duration of the pulse sent from the transmitter is greater than the duration of the pulses across the load resistor R_{L1}, and smaller

than those across $R_{1,2}$, then the pulse from the differentiating circuit coincides with the pulse of the distributor's first channel, which corresponds to the transmission of the first command, etc. The coincidence stages can be circuits designed with diodes, transistors, etc. An output seal-in relay is connected, if required, to the output of each stage.

The input pulses can be produced not only by a pulse generator in the transmitter (Fig. 3) but also by a pulse generator in the receiver, similarly to the way it is done in the VST-1 telemetry system [3]. In many cases, for example, in the use of frequency channels, it is necessary to have generators of primary pulses, synchronized by well-known means, both in the transmitter and receiver.

Fig. 4.

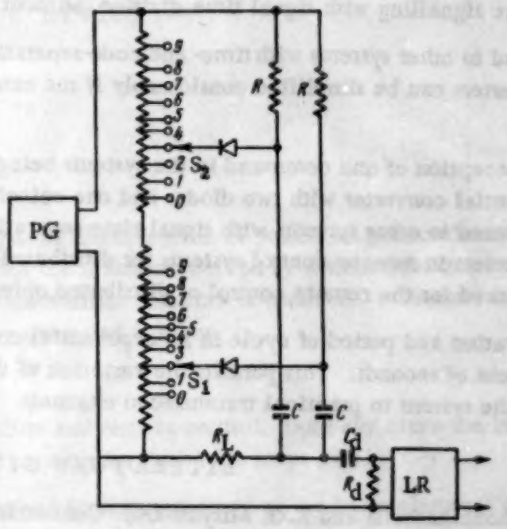

Fig. 5.

The maximum number of transmitted commands with a distributing code is

$$n = K \frac{t_{c\,max}}{\Delta t_{av}} - 1, \qquad (3)$$

where $\Delta t_{av} = (\Delta t_{min} + \Delta t_{max})/2$, K is a factor of assurance (K ≈ 0.7-0.8). The number of commands n, calculated from Eq. (3), can be increased several times if special forming circuits are introduced.

It should be noted that multichannel, signal time-separation systems formed by the selection of different levels of trigger voltages, were repeatedly described in the literature, while the mechanical analog of such systems was already realized in Prof. P.A. Molchanov's radiosonde. Thus, for example, in the circuit proposed by F.E. Temnikov, the separation of signals was achieved by the use of a special sawtooth generator, and a "bunch" of null-elements and rheostats connected in series [4].

However, such systems had relatively complex null-elements, auxiliary sawtooth generators, and other equipment with insufficiently stable parameters. In contrast to this, circuits with exponential converters do not contain unstable components which significantly affect the system's operation. Thus, the use of exponential converters permits one to realize simple and very stable, remote-control systems.

Figure 5 shows the circuit of a transmitter for a remote-control system with a decimal code, designed for the transmission of 100 commands. It contains two RC networks, two diodes and, correspondingly, two switches S_1 and S_2, designed for setting up tens and units. For the transmission of, say, order No. 67 switch S_1 sets at position 6 and switch S_2 sets at position 7.

The output of the exponential converter is connected to a differentiating network C_dR_d for sending pulses into the line when the first and second diodes open. The principle of pulse separation here is the same as in a multichannel, telemetry system using exponential converters [3].

The receiving unit of a remote-control system must have a distributor with individual outputs and coincidence circuits for each ten and unit, designed like, for example, the receiver in Fig. 3.

The circuits of the transmitters shown in Figs. 3 and 5 can transmit during one cycle only one command. For transmitting two or more commands during one cycle a correspondingly larger number of RC networks is introduced in the transmitter, for example, as is shown in Fig. 5.

SUMMARY

1. Exponential converters permit the production of simple and very reliable, contactless systems of remote control and remote signalling with signal time division, without the use of components with ferromagnetic cores.

2. Compared to other systems with time- and code-separation of signals, the circuit of the transmitter with exponential converters can be simplified considerably if the number of commands transmitted during one cycle is decreased.

3. For the reception of one command in the systems being considered the receiver's distributor can have a simplified exponential converter with two diodes and one coincidence circuit. This simplifies considerably the receiver, as compared to other systems with signal time-separation, and opens up wide areas of application for exponential converters in remote-control systems for distributed objects. Frequency systems of remote control, which are widely used for the remote control of distributed objects [2] possess this same property.

4. Pulse duration and period of cycle in an exponential converter can be chosen in the range of tens of microseconds to tens of seconds. This permits the variation of the system's response over a wide range, and simplifies matching the system to practical transmission channels.

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ANALYSIS OF THE OPERATION OF PULSED MAGNETIC ELEMENTS IN CONTACTLESS REMOTE-CONTROL DEVICES*

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This article provides an analysis of the operation of pulsed magnetic elements with rectangular hysteresis loops when the elements are fed by a sinusoidal voltage of industrial frequency. A series of characteristic relations is deduced. A method for calculating a single element and a pulse relay is given.

Among the contactless devices of automation and remote control, more and more use is being made of pulsed elements, with magnetic cores that have rectangular hysteresis loops (RHL).

Magnetic elements with RHL together with semiconductors serve as the basic elements of the contemporary contactless devices used in automation and remote control. They are used in the various loops of remote-control devices as pulse amplifiers or repeaters, memory elements, and so on.

Magnetic pulse elements with a large amount of positive feedback possess the characteristics of contactless polarized relays [1]. Such elements are used as output and intermediate relays.

It is possible to study two groups of circuits composed of pulsed magnetic elements. The first group consists of circuits which have series-connected loads, these are known as choke circuits [2]. The second group consists of circuits which have parallel-connected loads, these are known as transformer-type circuits [3-6].

The elements of the second group have several advantages over those of the first group. First of all, there is the ability to withstand large oscillations in the input voltage. It is because of this that the elements of the second group are used, in preference to those of the first group, in applications which take place during the development of remote-control devices. It is for this reason, that the present article will be devoted only to the study of pulse circuits with transformer-connected loads.

Of all the possible methods of feeding such elements, the simplest and most reliable seems to be that of using an alternating voltage of industrial frequency. In spite of this, this method of supply leads to a relatively large size of circuit element. The core size of the pulsed magnetic elements now in use depends mainly upon the frequency and form of the input pulses.

When building large-capacity devices, where a decrease in the size of the elements to be used in the device is significant, it is expedient to raise the frequency of the input pulses, and also to make use of special pulse shaping devices. However, for devices of relatively small capacity, for which the question of size is not of basic importance, the first consideration becomes that of achieving the maximum simplicity and reliability in the pulse supply.

These considerations determine the choice of the type of impulse generator to be used with a device of relatively small capacity (20-30 objects) operating on industrial frequency ac. The half-waves of the sinusoidal

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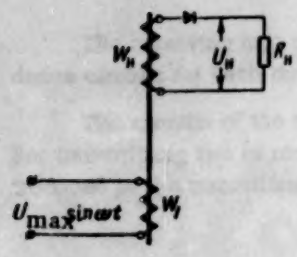

Fig. 1. Simplified circuit diagram of a magnetic element with sinusoidal input.

ac voltage may be used as the input pulses for the device. The operation of elements possessing RHL under these conditions is characterized by definite specifications and it differs from the operating conditions that exist when the circuit is supplied by current pulses, as is the case in computer techniques.

When elements with RHL are fed from a half-wave alternating voltage, the stable operation of these elements requires that there be a sufficiently rapid and complete magnetic polarity reversal of the core magnetization in order that the development of the necessary signal power at the output elements be insured. In connection with this, the time required for complete polarity reversal of the core magnetization t_{mag} may take on various values within the half-period of the supply voltage. The amplitudes and the effective values of the voltage and power of the output signals depend upon the magnetization time. In its turn the time required for a complete magnetic polarity reversal is determined by the amplitude of the input voltage and by the parameters of the magnetic element. With a given magnitude of output signal, the length of the time interval that is required for complete magnetic polarity reversal of the core can be determined, and from this we can calculate all the parameters of the magnetic pulse element. Therefore, in order to analyze and calculate the values of these elements, it is necessary to determine the dependence of the output voltage and power upon the time required for a complete reversal of the magnetic polarity of the core when the elements are fed by an alternating sinusoidal voltage.

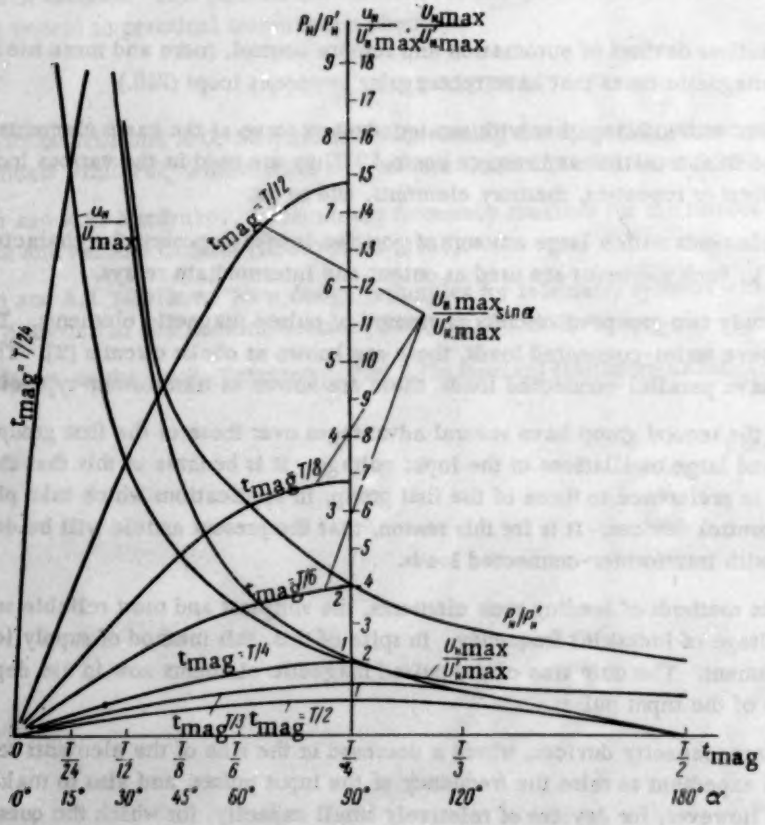

Fig. 2. Graph of the dependence of $U_{H\,max}/U'_{H\,max}$, $u_H/U'_{H\,max}$, P_H/P'_H upon the magnetization time (t_{mag}).

We will now turn our attention to the determination of this dependence. While doing this, we will approximate the actual hysteresis loop by a rectangular loop, and we will make use of the fact that the integral of the electromotive force existing in the winding taken over the time interval t_{mag} required for a complete reversal of the magnetic polarity of the core is a constant, and is independent of the magnetization time:

$$\int_0^{t_{mag}} e\,dt = -2B_r W S_A \cdot 10^{-8}, \tag{1}$$

where e represents the electromotive force in the winding being examined, B_r represents the residual induction, taken as being equal to the saturation value, W is the number of turns in the winding and S_A is the transverse cross section of the core.

This expression permits us to determine a series of relations characterizing the operation of magnetic elements for various magnetization times, and allows us to simplify the procedures required to calculate these elements. In order to determine the indicated relations let us examine the simplified circuit of a magnetic element with RHL, as shown in Fig. 1. We will neglect the resistance of the winding and the rectifiers.

I. The determination of $U_{H\max} = f_1(t_{mag})$, where $U_{H\max}$ represents the maximum value of the voltage (or the electromotive force) in the load windings when the core is not saturated.

Applying Eq. (1) to the load winding, we get

$$\int_0^{t_{mag}} u_H\,dt = -2B_r W_H S_A \cdot 10^{-8}. \tag{1a}$$

On the other hand, since the input voltage is sinusoidal we get

$$\int_0^{t_{mag}} u_H\,dt = -\frac{U_{H.\max}}{\omega}(1 - \cos\omega t_{mag}).$$

If we let $2B_r W_H S_A \cdot 10^{-8} = m$, we may write

$$U_{H.\max} = \frac{m\omega}{1 - \cos\omega t_{mag}}. \tag{2}$$

Let us designate by $U'_{H\max}$ the value of $U_{H\max}$ for the case where $t_{mag} = T/2$. Then, from (2), we get

$$U'_{H\max} = 0.5\,m\omega. \tag{2a}$$

Dividing (2) by (2a) we find:

$$\frac{U_{H\max}}{U'_{H\max}} = \frac{2}{1 - \cos\omega t_{mag}}. \tag{3}$$

II. The determination of $U_H = f_2(t_{mag})$, where U_H is the effective value of the electromotive force upon the load:

$$U = \sqrt{\frac{1}{T}\int_0^{t_{mag}} u_H^2\,dt} = U_{H\max}\sqrt{\frac{1}{T}\left(\frac{t_{mag}}{2} - \frac{1}{4\omega}\sin 2\omega t_{mag}\right)}.$$

Substituting the value of $U_{H\max}$ obtained in (2), we obtain

$$U_H = \frac{m\omega}{1 - \cos\omega t_{mag}}\sqrt{\frac{1}{T}\left(\frac{t_{mag}}{2} - \frac{1}{4\omega}\sin 2\omega t_{mag}\right)} \tag{4}$$

III. The determination of $u_H = f_3(t_{mag})$, where u_H is the instantaneous value of the load voltage at the end of the magnetization time, i.e., for $t = t_{mag}$.

Using the fact that $u_H = U_{H\max} \sin \omega t_{mag}$, in Eq. (3) we obtain the expression

$$\frac{u_н}{U'_{н\max}} = \frac{2 \sin \omega t_{mag}}{1 - \cos \omega t_{mag}}. \tag{5}$$

IV. The determination of $P_H = f_4(t_{mag})$, where P_H is the average load power.

Using Eq. (4) we obtain

$$P_н = \frac{U_н^2}{R_н} = \frac{(m\omega)^2}{R_н T (1 - \cos \omega t_{mag})^2} \left(\frac{t_{mag}}{2} - \frac{1}{4\omega} \sin 2\omega t_{mag} \right). \tag{6}$$

Let us designate by P'_H the value of P_H when $t_{mag} = T/2$. Then from (6) we obtain

$$P'_н = \frac{(m\omega)^2}{16 R_н}. \tag{6a}$$

Dividing (6) by (6a) we get

$$\frac{P_н}{P'_н} = \frac{16 \left(\frac{t_{mag}}{2} - \frac{1}{4\omega} \sin 2\omega t_{mag} \right)}{T (1 - \cos \omega t_{mag})^2}. \tag{6b}$$

Graphs of the expressions (3), (5) and (6b) are shown in Fig. 2.

With the aid of expressions (1)-(6) it is easy to analyze and to simplify the calculation of magnetic pulse elements under various operating conditions.

The basic element of contactless remote-control devices consists of the circuit of a single element with resistive load and without feedback paths (Fig. 3). Therefore, before going to the calculation of pulse relays, we must first stop and calculate the operation of this type of element. While we are doing this, let us agree to call the magnetization of the core due to the input impulse the "operation" of the element, and the time required for this we shall designate as t_{prep}.

Methods of Calculation of a Magnetic Pulsed Element with Resistive Load

We shall consider the reverse resistance of all rectifiers as being infinite. We shall only take account of the forward resistance of a rectifier when it occurs in the preparatory circuit, where its magnitude has a significant value.

For the load circuit, from Fig. 3,

$$U_H = E_H - I_H R_{wH}, \tag{7}$$

where E_H, U_H and I_H represent the effective values of the electromotive force, voltage and current.

The voltage drop in the load windings may be represented by

$$I_н R_{w.н} = \Delta q \rho \frac{l_w}{q} = 170 \cdot 10^{-6} \Delta l_н W_н, \tag{7a}$$

where Δ is the current density in the load winding in amp/mm^2, l_H is the average length of one turn of the load winding in cm and $170 \cdot 10^{-6}$ is the specific resistance of copper at 20°C in ohm-mm^2/cm.

Substituting for the value of $I_H R_{wH}$ in (7) the expression found in (7a), and for the value of E_H in (7) the expression found in (4), we obtain

$$U_н = \frac{m\omega}{1 - \cos \omega t_{op}} \sqrt{\frac{1}{T} \left(\frac{t_{cp}}{2} - \frac{1}{4\omega} \sin 2\omega t_{op} \right)} - 170 \cdot 10^{-6} \Delta l_н W_н.$$

Solving the last equation for W_H, with the appropriate substitution for m we get:

$$W_{\text{н}} = \frac{U_{\text{н}} \cdot 10^8}{\frac{0{,}2\,B_r S_{\text{A}}\,\omega}{1-\cos\omega t_{\text{op}}} \sqrt{\frac{1}{T}\left(\frac{t_{\text{op}}}{2} - \frac{1}{4\omega}\sin 2\omega t_{\text{op}}\right)} - 170\ \Delta l_{\text{н}}} \tag{8}$$

Using Eq. (8) we can determine the number of turns in the load winding required for any given value of t_{op}.

The preparatory pulses may be applied to the magnetic element by means of various remote-control devices. Figure 4 shows the circuit of the element I, the source of the preparatory pulses. The output winding of this element is represented by W_k, R_{B1} and R_{B2} represent the dynamic resistances of rectifiers 1 and 2, and R_r represents the resistance present in the preparatory circuit, its basic function is to limit the return information [4].

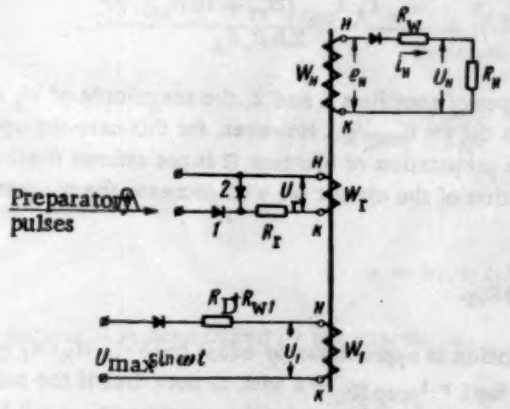

Fig. 3. Circuit of pulsed magnetic element (without feedback paths).

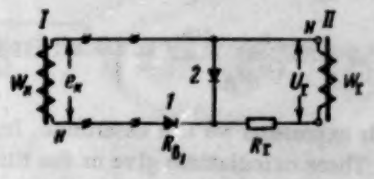

Fig. 4. Circuit of preparatory element.

The diagram given in Fig. 4 is the outline of a circuit which serves to transfer the current from core I, which can be considered the sending element, to core II which functions as the receiving element. This type of circuit is well known and has been described in the literature [4].

It has been shown in [4] that for cores I and II of the same size and with identical characteristics, the operation of the sending core must occur more slowly than the preparation of the receiving core, i.e., it is necessary to satisfy the conditions $t_{op\,I} > t_{prep\,II}$, where $t_{prep\,II}$ represents the time required to prepare element II.

As a preliminary step, let us assume that $t_{op\,I} = t_{prep\,II}$. In this case, during the preparation of element II of the preparatory circuit, we have

$$\int_0^{t_{op\,I}} e_{\text{н}}dt = \int_0^{t_{op\,I}} u_{B1}dt + \int_0^{t_{op\,I}} i_{\text{н}}R_r dt + \int_0^{t_{op\,I}} u_r dt. \tag{9}$$

Using the piecewise-linear approximation for the volt-ampere characteristic of rectifier 1, we get

$$u_{B1} = u_{ar\,1} + i_k R_{B1},$$

where $u_{ar\,1}$ is the threshold voltage in the forward direction of rectifier 1, R_{B1} is the dynamic resistance in the forward direction of rectifier 1, and i_k is the current resulting from the electromotive force e_k.

Substituting the value of u_{B1} in (9) and integrating, we get

$$2B_r S_{\text{A}} W_{\text{н}} \cdot 10^{-8} = u_{a\,r\,1} t_{op\,I} + \frac{H_c\, l_{\text{A}}(R_{B1} + R_r)}{0{,}4 + \pi W} t_{op\,I} + 2B_r S_{\text{A}} W_r \cdot 10^{-8}.$$

Applying the known condition $W_k = 2W_r$, and solving the latter equation for W_r, we get

$$W_r = \frac{u_{ar1}\, t_{op} \cdot 10^8}{4B_r S_A} \pm \sqrt{\frac{1}{4}\left(\frac{u_{r1}\, t_{op1} \cdot 10^8}{2B_r S_A}\right)^2 + \frac{H_c\, l_A (R_{B1} + R_r)\, t_{op} \cdot 10^8}{0{,}8\,\pi B_r S_A}}. \qquad (9a)$$

During the operation of element II the preparatory winding W_r may send the winding W_k return information which can partially prepare the sending element I, and in this manner lead to the presence of conditions which produce interference.

Let us assume that there is a tenfold weakening of the voltage applied to the W_k winding when the second element operates. This condition is approximated by the relations in $i_r R_r = 10u_{R_2} = 10u_{ar2} + 10i_r R_{B2}$, where i_r represents the current flowing through resistance R_r. Neglecting the term $10u_{ar2}$, we obtain $R_r = 10R_{B2}$. Since $W_k = 2W_r$, then the ratio of the change in current due to partial preparation of the core as a result of the return of information to the change in current resulting from complete preparation is approximately equal to 1/20. After the substitution of $R_r = 10R_{B2}$ in (9a) we get

$$W_r = \frac{u_{ar1}\, t_{op} \cdot 10^8}{4B_r S_A} \pm \sqrt{\left(\frac{u_{ar1}\, t_{op1} \cdot 10^8}{4B_r S_A}\right)^2 + \frac{H_c\, l_A\, t_{op1} (R_{B1} + 10R_{B2}) \cdot 10^8}{2{,}5\, B_r S_A}}. \qquad (10)$$

From this expression we can determine, for given types of rectifiers 1 and 2, the magnitude of W_r and the resulting W_k. These calculations give us the limiting case $t_{op\,I} = t_{prep\,II}$. However, for this case the operation of the circuit shown in Fig. 4 is not stable, since complete preparation of element II is not assured should the supply voltage be lowered. In order to insure stable operation of the circuit we will decrease the magnitude of R_r, so that

$$R_r = (5\text{-}7)R_{B2}. \qquad (10a)$$

Then the ratio of the complete to the partial preparation is approximately equal to (5-7) W_k/W_r = 10-14, which is perfectly acceptable. This assures the condition $t_{op\,I} > t_{prep\,II}$. We wish to note that if the parameters of the preparatory circuit (R_r, W_r) are calculated on the basis of operation under the condition $t_{op\,I} = 0.5T$, then the circuit operation will be stable if $t_{op\,I} > 0.5T$.

As a matter of fact, it follows from Eq. (1) that

$$\int_0^{t_{opI}} e_k dt \equiv S = \text{const},$$

where S is the area bounded by the curve e_k and the x-axis, as shown in Fig. 5. Therefore, we may write, that for $t_{op} \leq 0.5T$

$$S_1 + S_2 = S'_1 + S'_2 = S,$$

where the areas S_1 and S'_1 are proportional to the change in inductance of the element II, in the preparatory process of reversing the magnetic polarization of element I, during the time $t_{op\,I} = 0.5T$, where $t_{op\,I} < 0.5T$, and the areas S_2 and S'_2 are proportional to the energy absorbed by the resistor R_r during the time $t_{op\,I} = 0.5T$, where $t_{op\,I} < 0.5T$.

It is clear from Fig. 5 that $S'_2 < S_2$ and therefore $S'_1 > S_1$. Therefore, we see that if the area S_1 is to be of sufficient size to insure the complete preparation of element II, then the area S'_1 will be more than adequate to insure the complete preparation of this element.

The current in the input winding W_1 (Fig. 3) is a maximum when the element is in the "nonprepared" state, i.e., when the preparatory pulse is absent and the inductive reactance of winding W_1 is practically zero. For this case, the rectified current in the input circuit is given by

$$I_1 \approx \frac{U}{R_D} = \frac{U_{max}}{2R_D},$$

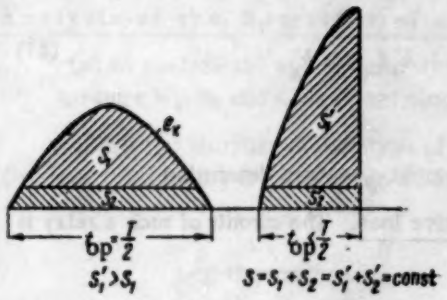

Fig. 5. Curves of the voltage in the preparatory circuit during the time interval $t_{op} = 0.5T$, where $t_{op} < 0.5T$.

where R_D is the additional resistance in the input circuit and U is the effective value of the sinusoidal voltage applied to the input circuit.

Given the diameter of the conductors of the input winding W_1, we can determine the value of the current I_D so that we will not exceed it.

It is obvious that the condition

$$I_1 \approx \frac{U_{max}}{2R_D} \leqslant I_D \tag{11}$$

must be satisfied.

Using the above condition we can determine the additional resistance

$$R_D \geqslant \frac{U_{max}}{2I_D} = \frac{\sqrt{2}\,U}{2I_D}. \tag{12}$$

For the input circuit

$$u = u_1 + i_1 R_D + i_1 R_{w1}. \tag{13}$$

The current i_1 is determined by the expression

$$i_1 = i_0 - \frac{W_n}{W_1} i_n - \frac{W_r}{W_1} i_r \tag{14}$$

Substituting the value of i_1 from (14) into (13), and integrating both parts of the resulting expression between the limits 0 and t_{op} (operating time of the element), we get:

$$\int_0^{t_{op}} u\,dt = \int_0^{t_{op}} u_1\,dt + \int_0^{t_{op}} i_0 R_D\,dt - \int_0^{t_{op}} \frac{W_n}{W_1} R_D i_n\,dt - \int_0^{t_{op}} \frac{W_r}{W_1} R_D i_r\,dt +$$
$$+ \int_0^{t_{op}} R_{w1} i_0\,dt - \int_0^{t_{op}} \frac{W_n}{W_1} R_{w1} i_n\,dt - \int_0^{t_{op}} \frac{W_r}{W_1} R_{w1} i_r\,dt. \tag{15}$$

Performing the integration, we get

$$\frac{\sqrt{2}\,U}{\omega}(1 - \cos\omega t_{op}) = 2B_r S_A W_1 \cdot 10^{-8} + \frac{\sqrt{2}\,UH_c\,l_A\,t_{op}}{0{,}8\,\pi W_1 I_D} +$$
$$+ \frac{\sqrt{2}\,W_n^2 UB_r S_A \cdot 10^{-8}}{W_1\left(R_n + 1{,}7\cdot 10^{-4}\frac{l_n}{d_n} W_n\right)} + \frac{\sqrt{2}\,UW_r^2 B_r S_A \cdot 10^{-8}}{8W_1 R_{n2} I_D} + \frac{1.7\cdot 10^{-4} H_c\,l_A\,l_1\,t_{op}}{0.4\,\pi d_1} +$$
$$+ \frac{3{,}4\cdot 10^{-12} W_n^2\,l_1 B_r S_A}{d_1\left(R_n + 1{,}7\cdot 10^{-4} W_n \frac{l_n}{d_n}\right)} + \frac{1.7\cdot 10^{-12} W_r^2\,l_1 B_r S_A}{4R_{n2} d_1} \tag{16}$$

As a result of Eq. (16) we will consider the magnetizing current constant and equal to $H_c l_A / 0.4\pi W_1$ for the time $t = 0$ to $t = t_{op}$.

Equation (16) may be placed in the form

$$kU(1-\cos\omega t_{op}) = k'_1 W_1 + k'_2 \frac{t_{op}}{W_1} + k'_3 \frac{U}{W_1} + k'_4 t_{op} + k'_5, \qquad (17)$$

where the coefficients k'_1, k'_2, k'_3, k'_4 and k'_5 are constants.

Using Eq. (17) we can determine W_1 given U and t_{op}, or given W_1 and t_{op} we can determine U.

Calculation of the parameters of a two-core pulse relay with resistive load. The circuit of such a relay is shown in Fig. 6.

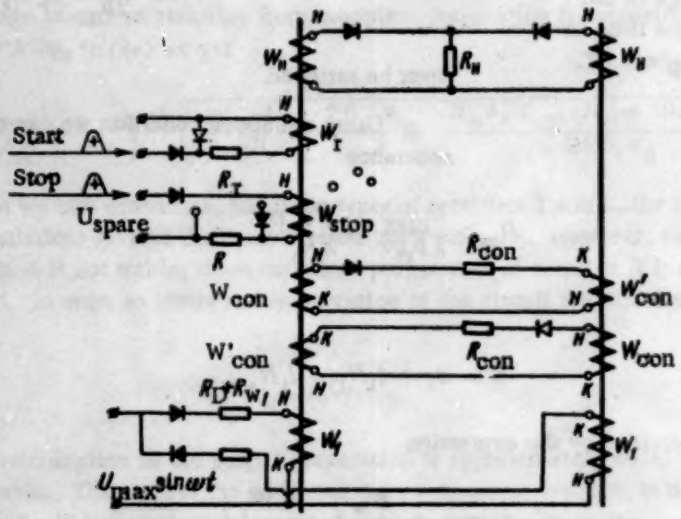

Fig. 6. Circuit of a two-core (RHL) pulse relay.

The method derived above may, with a few corrections, be used for the calculation of the given relay.

The number of turns in the load winding of each core is given by the expression

$$W_H = \frac{U_H \cdot 10^6}{\frac{6{,}28 B_r S_A}{1-\cos\omega t_{op}} \sqrt{\frac{2}{T}\left(\frac{t_{cp}}{2} - \frac{1}{4\omega}\sin 2\omega t_{op}\right) - 170\,\Delta l_H}}.$$

We see that in this case the expression in parentheses, under the square-root sign, is multiplied by the factor 2/T, rather than by the factor 1/T, as was the case for the element without return paths. This is due to the fact that current flows in the load winding during both halves of the input voltage cycle.

The number of turns in the start winding W_r and the resistance R_r are determined, as before, from Eqs. (10) and (10a).

In order to determine the number of turns in the intermediate connecting windings W_{con} and W'_{con}, we may use Eq. (9a) with R_{con} substituted for R_r.

By analogy with the case described above, it is necessary, in order to assure stable operation of the relay, to choose a value for the resistance R'_{con} that is somewhat lower than that used in calculating W_{con} and W'_{con}: $R'_{con} = (0.5\text{-}0.7)R_{con}$.

In order to determine the input parameters R_D, U and W_1 of the relay we may use Eq. (12), and an expression analogous to (17) may be used to take into account the supplementary effects due to the ampere-turns of the intermediate connecting winding.

Analysis of the Operation of a Single-Core Pulse Relay with Resistive Load

Let us analyze the operation of a relay with a resistive load, the circuit of the relay being represented by Fig. 7, where W_2, R_2 and C represent elements of the feedback path.

In order to simplify the analysis of the relay operation we will neglect the forward resistance of the rectifiers and the winding resistance. In addition to this, we will also neglect the influence of the start and stop circuits. These assumptions are permissible since the influence of the indicated factors upon the operation of the relay is negligible.

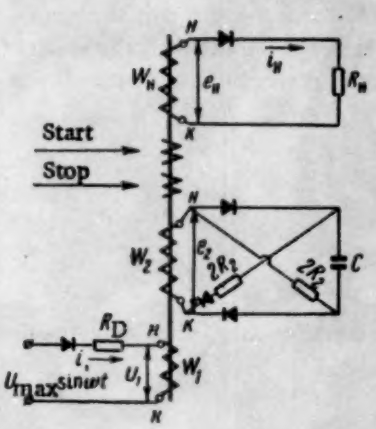

Fig. 7. Circuit of a single-core (RHL) pulse relay.

During the operating half-period

$$i_0 = i_1 + \frac{W_2}{W_1} i_2 + \frac{W_н}{W_1} i_н, \tag{18}$$

where

$$i_2 = i_R + i_C, \quad i_C = C\frac{du_C}{dt},$$

$$i_R = \frac{e_2}{R_2}, \qquad i_н = \frac{e_н}{R_н}.$$

We have found, by means of experiments, that the most stable operation occurs in the case where the relay operates during the time $t_{op} \leq 0.25T$, i.e., $\omega t \leq 90°$.

Then during the time that the relay is in the operated condition, $e_2 = u_C$ and therefore,

$$i_1 = i_0 - \frac{W_2}{W_1}\frac{e_2}{R_2} - C\frac{W_2}{W_1}\frac{de_2}{dt} - \frac{W_н}{W_1}\frac{e_н}{R_н}. \tag{19}$$

The following relation holds for the relay input circuit

$$u = i_1 R_D + u_1. \tag{20}$$

Substituting the value of i_1 obtained from Eq. (19) into Eq. (20) and making use of the relations

$$e_2 = -\frac{W_2}{W_1}u_1, \quad e_н = -\frac{W_н}{W_1}u_1, \quad k_2 = \frac{W_2}{W_1}, \quad k_н = \frac{W_н}{W_1},$$

we obtain the differential equation

$$\frac{du_1}{dt} + \frac{R_D R_н k_2^2 + R_D R_2 k_н + R_2 R_н}{R_D R_2 R_н C k_2^2} u_1 - \frac{u}{k_2^2 R_D C} + \frac{i_0}{k_2^2 C} = 0. \tag{21}$$

Solving this differential equation for the initial conditions $u_1 = 0$ and $t = 0$, we get

$$u_1 = \frac{U_{max}}{k_2^2 C R_D}\left(\frac{p}{p^2+\omega^2}\sin\omega t - \frac{\omega}{p^2+\omega^2}\cos\omega t\right) + \\ + \left[\frac{U_{max}\,\omega}{k_2^2 C (p^2+\omega^2) R_D} + \frac{i_0}{k_2^2 p}\right]\varepsilon^{-pt} - \frac{i_0}{C k_2^2 p}, \tag{22}$$

where

$$p = \frac{R_D R_н k_2^2 + R_D R_2 k_н + R_2 R_н}{R_D R_2 R_н C k_2^2}.$$

After some manipulation Eq. (22) may be placed in the form

$$u_1 = \frac{U_{max}}{k_2 C R_D \sqrt{p^2+\omega^2}} \sin\left(\omega t - \operatorname{arc\,tg}\frac{\omega}{p}\right) + \left[\frac{U_{max}\,\omega^2}{k_2^2 C R_D (p^2+\omega^2)} + \frac{i_0}{C k_2^2 p}\right]\varepsilon^{-pt} - \frac{i_0}{C k_2^2 p}. \quad (23)$$

We see from Eq. (23) that the voltage in the relay winding consists of three parts, the first part is a sinusoidal voltage of the same frequency as the input voltage but with a different phase and amplitude. The second part contains the factor ϵ^{-pt} and represents an exponential voltage. The third part represents a constant voltage and is the result of the magnetizing current in the core.

During the operating period of the relay (Fig. 8)

$$\int_0^{t_1} u_1 dt = 2 B_r S_A W_1 \cdot 10^{-8}.$$

If we substitute the value of u_1 from Eq. (22) into the above equation the equation may, after some manipulation, be placed in the form

$$\left[\frac{U_{max}\,\omega}{k_2^2 C R_D (p^2+\omega^2)\,p} + \frac{i_0}{C k_2^2 p^2} - \frac{i_0 t}{C k_2^2 p}\right] - \left[\frac{U_{max}\,\omega}{p k_2 C R_D (p^2+\omega^2)} + \frac{i_0}{C k_2^2 p^2}\right]\varepsilon^{-pt} -$$

$$- \left[\frac{U_{max}\,p\cos\omega t_1}{k_2^2 C R_\Gamma (p^2+\omega^2)\,\omega} + \frac{U_{max}\,\omega\sin\omega t_1}{k_2^2 C R_D (p^2+\omega^2)}\right] = 2 B_r S_A W_1 \cdot 10^{-8}. \quad (24)$$

At the end of the magnetization (operation) the voltage across the capacitor increases to the value u_{C1} (Fig. 8). After this the current changes in the core and consequently the capacitor begins to discharge through the winding W_2.

The discharge current tries to prepare the relay for its next operation but, since current is still flowing in the input winding W_1 up to the time $t_2 = 0.5T$, and the ampere-turns $i_1 W_1 > i_{C\,disch} W_2$, this preparation does not occur. The inductive reactance of the winding W_2 is zero for the time $t_2 - t_1$, and the discharge time constant is $R_2 C$. During this time, the capacitor voltage decreases, following the rule

$$u_C = e_{21}\varepsilon^{-\frac{t-t_1}{4R_2C}} \quad (25)$$

For $t = t_2$

$$u_{C2} = e_{21}\varepsilon^{-\frac{t_2-t_1}{4R_2C}}, \quad (26)$$

where e_{21} represents the electromotive force in the feedback winding during the capacitor discharge for $t = t_1 = t_{op}$.

At the time $t = t_2$ the current i_1 ceases, and the winding "becomes part of" the discharge circuit, from this time on and until the polarity of the core is completely reversed, the discharge current will be constant.

In order to insure stable operation of the relay during the changes in the input voltage (decreases), provision must be made earlier to insure an adequate voltage supply. With this aim in mind, we require that the core return to its initial state during the time $t_3 - t_2 = \frac{1}{3}T$, where $t_3 < t_4 = T$.

From Fig. 8a we get

$$U_{K1} = U_{C2} - 4 i_{mag} R_2, \quad U_{K2} = U_{K1} - \Delta U_C;$$

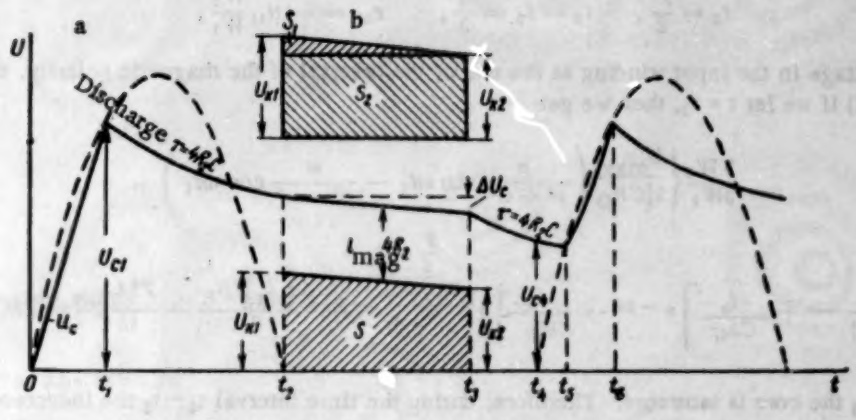

Fig. 8. Voltage curves for the feedback circuit.

where i_{mag} represents the magnetizing current, which is equal to the capacitor discharge current during the return of the capacitor to its initial state.

$$\Delta U_C = \frac{1}{C}\int_{t_2}^{t_3} i_{mag}\,dt = i_{mag}(t_3 - t_2)/C.$$

Since we have assumed that $t_3 - t_2 = {}^1/_3 T$, then $\Delta U_C = i_{mag}T/3C$. In order that the element be fully prepared during the time $t_3 - t_2$, it is necessary that

$$\int_{t_2}^{t_3} u_2\,dt = S = \frac{U_{k2} + U_{k1}}{2}(t_3 - t_2). \tag{27}$$

The latter expression becomes clearer if we refer to Fig. 8b, where the area S represents the number of volt-seconds required for the complete reversal of the magnetic polarization of the core.

Using the basic Eq. (1) we get

$$\int_{t_2}^{t_3} u_2\,dt = 2B_r S_A W_2 \cdot 10^{-8}.$$

Thus,

$$\frac{U_{k2} + U_{k1}}{2}(t_3 - t_2) = 2B_r S_A W_2 \cdot 10^{-8}.$$

After substituting the values of U_{k1} and U_{k2} and rearranging the resulting expression, we get

$$(t_3 - t_2)\,e_{21}\varepsilon^{-\frac{t_2 - t_1}{4R_1C}} - 4R_2\,i_{mag}(t_3 - t_2) - \frac{i_{mag}(t_3 - t_2)^2}{2C} = 2B_r S_A W_2 \cdot 10^{-8}, \tag{28}$$

from which

$$e_{21} = \frac{2B_r S_A W_2 \cdot 10^{-8} + 4R_2 i_{mag}(t_3 - t_2) + \frac{i_{mag}(t_3 - t_2)}{2C}}{t_3 - t_2}\,\varepsilon^{\frac{t_2 - t_1}{4R_1C}}.$$

If, in the latter equation, we let $t_3 = T$, then we get the minimum value of the electromotive force in the blocking winding that is necessary for the complete preparation of the core.

If, in Eq. (28), we make the substitution shown below

$$t_2 = \frac{T}{2}, \quad t_3 - t_2 = \frac{T}{3}, \quad e_{21} = -u_{11}\frac{W_2}{W_1},$$

where u_{11} is the voltage in the input winding at the end of the reversal of the magnetic polarity, this may be obtained from Eq. (22) if we let $t = t_1$, then we get

$$\frac{TW_2}{3W_1}\left\{\frac{U_{max}}{k_2^2 CR_D}\left(\frac{p}{p^2+\omega^2}\sin\omega t_1 - \frac{\omega}{p^2-\omega^2}\cos\omega t_1\right)+\right.$$

$$\left.\left[\frac{U_{max}\omega}{k_2^2 CR_D(p^2+\omega^2)} + \frac{i_0}{Ck_2 p}\right]\varepsilon^{-pt} - \frac{i_0}{Ck_2^2 p}\right\}\varepsilon^{-\frac{\frac{T}{2}-t_1}{4R_2C}} - \frac{4\,i_{mag}TR_2}{3} - \frac{T^2 i_{mag}}{18C} = 2B_r S_A W_2 \cdot 10^{-8}.$$

At the time t_3 the core is saturated. Therefore, during the time interval $t_4 - t_3$ the inductance of the W_2 winding will be zero. As a result, the capacitor will continue to discharge with time constant $4R_2C$. The voltage across the capacitor during the discharge time $t - t_3$ is given by the equation

$$u_C = U_{C3}\varepsilon^{-\frac{t-t_3}{4R_2C}} = U_{C3}\varepsilon^{-\frac{t-\frac{5T}{6}}{4R_2C}}.$$

Since

$$U_{C3} = U_{C2} - \Delta U_C = e_{21}\varepsilon^{-\frac{\frac{T}{2}-t_1}{4R_2C}} - \frac{i_{mag}T}{3C}.$$

and, therefore,

$$u_C = \left(e_{21}\varepsilon^{-\frac{\frac{T}{2}-t_1}{4R_2C}} - \frac{i_{mag}T}{3C}\right)\varepsilon^{-\frac{t-\frac{5T}{6}}{4R_2C}}, \tag{30}$$

then at time $t = t_4$, we have

$$u_{C4} = \left(e_{21}\varepsilon^{-\frac{\frac{T}{2}-t_1}{4R_2C}} - \frac{i_{mag}T}{3C}\right)\varepsilon^{-\frac{t_4-\frac{5T}{6}}{4R_2C}}. \tag{31}$$

After the time $t = t_4$, the second input pulse appears and the relay operates, Fig. 8a. While this is occurring, and up to the time $t = t_5$, there is joint action of the input and blocking circuits, this action may be described by means of a differential equation of the first order, the solution of the equation consists of three terms as shown in Eq. (23). At the time $t = t_5$, the capacitor starts to charge, after which the process repeats itself, according to the description given above.

The expressions (23), (24), (29) and (31) may be used to determine a series of relay parameters.

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MACHINE ANALYSIS OF SWITCHING CIRCUITS

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Algorithms for the mechanization of the structural analysis of switching circuits and for the sequential determination of the action of their elements are presented. The realization of the algorithms in a special logical machine is described.* The machine performs a complete analysis of switching circuits of practically any complexity much better and more rapidly than man does. The results of the analysis are recorded manually or automatically. The machine may be used for the analysis of switching circuits with solid-state elements and for the solution of logical and other types of problems. The principles involved can be used for automatic quality control in the production of relay devices and other electrical apparatus.

1. Introduction

Machines composed of relays are widely used in all branches of technology and national economy, they are often extremely complex and a great number of qualified intellectual workers are occupied in the design of these machines and their maintenance after assembly. The mechanization of the design of relay machines would free these workers from many monotonous and tiresome operations, would sharply raise the productivity and quality of their work and would provide them with the possibility of investigating newer and more difficult problems of relay technology, the solutions for which, in spite of the great success and stormy development of the theory of relay machines, are not at the present time within their ability. Also, as an end result, the specialization of machines for the solution of the problems involved in the synthesis and analysis of relay devices would greatly influence the further development of science and technology.

Efforts have been made both here [1, 2, 3] and abroad [4, 5] to devise a machine that will facilitate and accelerate the process of switching circuit design. Papers [3, 5] which are devoted to the problem of the mechanization of the synthesizing process will not be examined here.

The switching-circuit analyzer of Shannon and Moore [4], has been designed by the authors for use in connection with the synthesis of two terminal switching circuits made up of the contacts of at most four relays. This analyzer allows us to determine whether the circuit being investigated satisfies the specified conditions of operation, it is simple and consists of a small amount of apparatus. The principles upon which the analyzer is based are not limited to two-terminal networks or to four relays, however, for example, each addition of one relay to the circuits being analyzed would double the size of the machine and the quantity of apparatus. The analyzer cannot be applied to relay circuits involving many contacts or to circuits involving the relay windings as well as the contacts.

Tsukanov's matrix analyzer [1] mechanizes part of the operation by matrix analysis of the switching circuits. The time required for the analysis depends upon the number of relay contact chains in the circuit being analyzed rather than upon the number of relays in the circuit as in the Shannon-Moore analyzer.

*This machine has been developed by the author under the direction of Professor M.A. Gavrilova.

The results obtained by use of the analyzer cannot be applied directly, without supplementary processing, to the design of a circuit in the general form* that fulfills the desired conditions. The investigation of multi-contact circuits by means of the matrix analyzer is inconvenient and also it is not sufficiently mechanized. Complete automation of the process of analysis is impossible because of the method that has been chosen to introduce the circuit under analysis into the machine (the structural formulas for the circuit chains are written in chalk, between the loops on the board located in the front of the analyzer. All of the three versions of the matrix analyzer that have been constructed (the one using electromagnetic relays, the one using step-by-step switches and the one using perforated cards) do not permit the analysis of a circuit involving more than six loops, and in no way differ from one another in the functions that they fulfill.

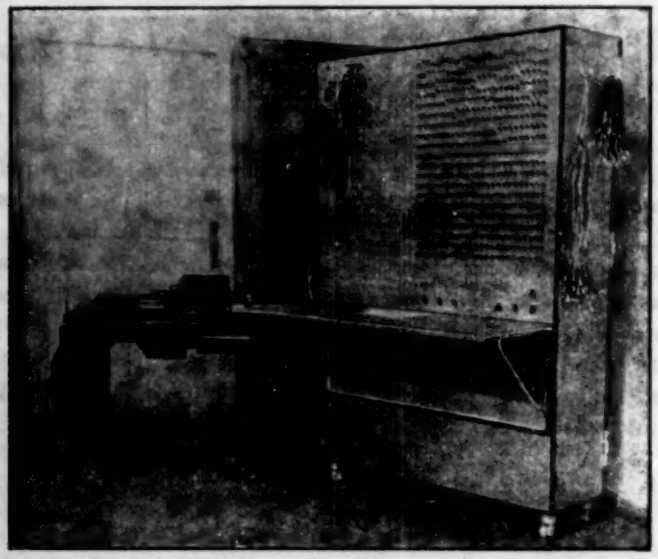

Fig. 1.

Rodin's electronic analyzer [2] is an electronic analog of the Shannon and Moore analyzer. The electronic analyzer has the capacity required for the analysis of a six-relay circuit. This analyzer is also the first fast-acting machine of its kind. While it is possible to increase the capacity of the analyzer, this is very difficult due to the resulting large increase in the size of the machine.

In contrast to the existing machines, the machine that is described in this article, the IAT AN SSSR, does not have the principal limitation, that of capacity, in addition it can work with both single-cycle or multiple-cycle circuits, and also with switching circuits of the general form that have already been assembled and built. The machine verifies the circuit operation, either for all combinatorial states of the circuit elements, or for certain definitely designated circuit combinations, for multiple-cycle circuits the machine indicates the sequential action of the circuit elements directly, and the results may either be recorded manually or they may be printed automatically by the machine. The machine may also be used to solve a number of other problems of a technical-logical character. The exterior view of one of the first models of the machine, together with the printing apparatus, is shown in Fig. 1.

A machine with the capacity for analyzing twenty element switching circuits consists of 180 relays RKN, six step-by-step switches of the type SHI-25/8,** 500 tri-positional switches and 120 signal lamps. The overall size of the machine is 1800 x 1200 x 350 mm. If we desire to increase the capacity of the machine so as to enable it to analyze the operation of switching circuits containing one more element, then it would be necessary

*By the general form of a circuit we mean a form in which the relay contacts (or other reacting organs of the circuit elements) are shown as part of either a shunt or series connection.

**Instead of the indicated step-by-step switches three reversible switches of the type RSHI-25/8 may be used.

to add eight more relays, two more mounting boards of switches and a proportionate amount of other apparatus.

We wish to note that it would simply not be realistic to build an analyzer of the type described in [4], that would have the capacity to analyze twenty-element switching circuits, since this would require switches with 1,048,576 contacts just to account for the states of the constituent elements, not to mention the rest of the apparatus that would be required.

Reference [6] gives a short description of the first version of the machine IAT AN SSSR that was constructed and its operating conditions. At the present time a more perfected type of machine, built in a more economical block version, has been developed and it is this machine that will be considered in the following discussion.

2. Mechanization of the Analysis of Circuit Structure

In the general case, switching circuits consist of receiving, intermediate and executive elements, each of which has reacting and performing organs. The problem involved in analyzing the structure of a switching circuit consists in determining the operating conditions of the reacting organs of its intermediate and executive elements and their dependence upon the state of the performing organs of the receiving and intermediate elements.

Let us designate the reacting organs of the various elements by capital letters, and their performing organs by the corresponding small letters. Let us use for the performing organs $\underline{p}$ of the receiving elements of the circuit, the designations $\tilde{a}_1, \ldots, \tilde{a}_i, \ldots, \tilde{a}_p$, where $\tilde{a}_i$ is the same as a_i or $\bar{a}_i$, let us use for the reacting organs $\underline{q}$ of the intermediate elements of the circuit, the designations $X_1, \ldots, X_j, \ldots, X_q$ and for the performing organs of the intermediate elements of the circuit, the designations $\tilde{x}_1, \ldots, \tilde{x}_j, \ldots, \tilde{x}_q$, and let us use for the reacting organs $\underline{r}$ of the executive elements of the circuit, the designations $Z_1, \ldots, Z_k, \ldots, Z_r$. In order to be definite, we wish to state that this paper will consider basically switching circuits that are constructed of two-position elements. However, taking into consideration the generality of application of relay circuits, the results obtained may be, to a great extent, applied to circuits composed of "yes-no" elements of any type.

When classified according to their principles of operation, switching circuits may be single-cycled (circuits of class O), in this case $q = 0$, or many-cycled (circuits of class M). From the structural point of view the circuits may be classified as parallel-series (circuits of class P) or circuits with bridged connections (circuits of class N).

In order to solve the problem of the mechanization of the structural analysis of switching circuits, it is necessary first of all that we determine a method of analysis that can be applied to circuits of all classes in the general form. The degree of formalization of the chosen method must permit the construction of an algorithm which permits a sufficiently simple realization of the problem in the machine. An investigation of the existing analytic, graphical, matrix and other methods of analysis shows that the most suitable method of analysis for our purpose is the method of resolving the circuit into its constituent units [7].

In the proposed form [8] this method is applied only to the structural analysis of two-terminal contact networks of Class II ($p > 0$, $q = 0$, $r = 1$). The method is based on the fact that any Boolean function $\underline{p}$ may be represented as the sum of terms, each of which consists of the product of all the variables $\underline{p}$ or their negatives, taken in various combinations:

$$f_k(a_1, \ldots, a_i, \ldots, a_p) = \sum_{l=0}^{2^p-1} h_l \prod_{i=1}^{p} \tilde{a}_i. \tag{1}$$

Here the a_i are the variables, the h_l are the coefficients of the variables, the coefficients assume the values 0 or 1, Σ is the summation sign and Π represents the product (multiplication).

Analysis of the conditions required for the operation of the two-terminal relay circuit by means of (1) leads to the equation, which sets forth the combinatorial state* of the receiving elements,

$$\prod_{i=1}^{p} \tilde{a}_i = \{\tilde{a}_1 \tilde{a}_2 \ldots \tilde{a}_i \ldots \tilde{a}_p\}, \tag{2}$$

*We will designate the combinatorial state of the circuit elements by means of the product of symbols as in (1) or (2).

for which a closed path exists between the terminals. These combinations correspond to $h_l = 1$ and represent the complete elementary chain of the two-terminal contact circuit. An analogous set to that of Eq. (2), corresponding to $h_l = 0$, gives the results of the analysis for nonoperation.

Extension of the analysis (1) to two-terminal relay contact networks of class N is based upon the fact that any such two-terminal network, when analyzed according to the initiating or terminating elements [7], may be converted to an equivalent two-terminal network of class P.

For given conditions of operation the elements of a circuit, which is of the general form, usually satisfy the formula [9]

$$F_{Z_k} = f[Z_k]\, f\{\overline{Z_k}\}, \tag{3}$$

where the Z_k represent the reacting organs of the elements being analyzed, $f[Z_k]$ represents the chain switched in as a result of the action of the Z_k, and $f\{Z_k\}$ is the inverse chain of parallel Z_k. Since the chains $f[Z_k]$ and $f\{Z_k\}$ represent two-terminal networks, then instead of (3) we may write

$$F_{Z_k} = \sum_{l=0}^{2^p-1} [h_l] \prod_{i=1}^{p} \tilde{a}_i \sum_{l=0}^{2^p-1} \overline{\{h_l\}} \prod_{i=1}^{p} \tilde{a}_i. \tag{4}$$

It is not difficult to prove that the conditions that are necessary for the operation of the element Z_k, require that we consider the set of elements $\tilde{a}_l$, for which

$$[h_l]\,\overline{\{h_l\}} = 1 \tag{5}$$

and the conditions that must be fulfilled for the nonoperation of the element Z_k, require that

$$[h_l]\,\overline{\{h_l\}} = 0. \tag{6}$$

The physical analysis of switching circuits by means of the division of the switching circuit into its constituent units depends upon the determination of the state of the relay windings and the state of the relay contacts. For this it is essential that changes in the state of the contacts be entirely independent of the state of the relay windings. This last condition is naturally fulfilled in all cases for circuits of class O, but is physically contradictory for circuits of class M.

If we analyze the circuits of class M, we see that it is possible to subdivide them into their constituent units only if during the analysis we turn our attention away from the existing dependence between the relay windings and the contacts of the intermediate relays. In other words, in the course of circuit analysis we cut the return paths that exist, for this is required to change the circuit to one of class O. Let us call the latter circuit, the single-cycle circuit equivalent of the original class M circuit (or, more briefly, the equivalent circuit). It is clear that the receiving elements of the equivalent circuit $\tilde{\alpha}_1, \ldots, \tilde{\alpha}_{p+q}$ are the receiving elements $\tilde{\alpha}_1, \ldots, \tilde{\alpha}_p$ and the performing organs $\tilde{x}_1, \ldots, \tilde{x}_q$ of the original circuit of class M; the performing elements of the equivalent circuit $\Psi_1, \ldots, \Psi_{q+r}$ serve as the reacting organs of the intermediate elements $X_1, \ldots, X_q$ and the performing elements $Z_1, \ldots, Z_r$ of the original circuit. We will call the receiving elements of the equivalent circuit the input elements and the performing elements of the equivalent circuit the output elements.

For the single-cycle equivalent circuit, Eq. (4) holds, provided the proper symbols and indices are substituted:

$$F_{\Psi_k} = \sum_{l=0}^{2^{p+q}-1} [h_l] \prod_{i=1}^{p+q} \tilde{\alpha}_i \sum_{l=0}^{2^{p+q}-1} \overline{\{h_l\}} \prod_{i=1}^{p+q} \tilde{\alpha}_i, \tag{7}$$

where $k = 1, \ldots, q + r$. The expansion of (7) holds for the most general case, for $q = 0$ we get (4), if the shunting chains are absent we get (1); $p = 0$ leads to the autonomous circuit (10), while $r = 0$ leads to a circuit without performing elements.

Fig. 2.

The mechanization of the analytic process assumes that the circuits to be analyzed are introduced into the machine in some manner. The method of introducing the circuit into the machine influences, to a significant degree the complexity of the algorithms realized by the machine. The method chosen for use with the machine described in this article, the introduction of a physical model of the circuit to be analyzed, is the most direct and natural method of introducing the required data, not only does it permit us to realize in a sufficiently simple manner the algorithm of the structural analysis and, as developed below, also an algorithm that enables us to determine the sequential action of the elements in a switching circuit of class M, but it also insures the undisputed pre-eminence of specialized machines, even those constructed of electro-mechanical apparatus, over the universal rapid-action computers for the investigation and analysis of switching circuits.*

The presentation of circuits of class M in the form of single-cycle equivalents is necessary in principle in order to model the circuit physically, it insures a uniform process of structural analysis that is independent of the class of circuit being analyzed, and it is also useful in connection with the investigation of questions concerning the determination of the sequential action of the circuit elements. More than this, an understanding of the operation of the single-cycle equivalent circuit permits us to apply all the methods of analysis and synthesis developed for circuits of class O to circuits of class M.

Taking into account the fact that the circuit of class O is, in the sense determined above, a particular case of the single-cycle equivalent circuit, we will, in the following analysis, speak in terms of the latter circuit.

The physical model of the circuit may differ to a significant degree from the relay circuit intended for industrial use in its construction, in the parameters of the apparatus used, the time scale, and other such factors. However the model must preserve all the circuit characteristics essential in any given case for the aims of the analysis. This, first of all, refers to the circuit structure, i.e., the relationships between the elements and the character of the electrical connections between them, and in many cases to the mutual parametric dependence that exists in circuits of the general form. The basic elements used in the modeling of switching circuits consist of relay contacts (make and break contacts) which take the part of input elements, the relay windings which are used in the capacity of output elements, and the conductors which provide the necessary electrical connections in accordance with the structure of the equivalent circuit. In addition to this, it is advisable that the model contain among its elements resistors and rectifiers. Of course the model must also possess a power supply.

Due to the lack of space we will not discuss such general questions as how to determine the capacity of the model, how to determine suitable parameters for the elements of the model, the design of the power supplies, the electrical connections, and other such details that may be involved in the modeling of switching circuits. We will limit ourselves here to a short description of the decisions that were made in the construction of the machine that is being described in this paper. The input elements of the circuits to be analyzed are represented in the model by the contacts of the receiving relays C_i, and the output elements are represented in the model by the windings of the performing relays K_i. The contacts and relay windings are connected to sockets which appear on the plug-in board of the machine. Two sockets are provided for each electrical element of the circuit being analyzed to facilitate the multipling of connections associated with each loop of the circuit. Each relay C_i is provided with three transfer, one break and two make contacts, and each relay K_i has three identical, separate windings. The machine is provided with an equal number of receiving and performing relays, the basic factor in determining its capacity. In this manner the windings of the C_i relays may control the action of the input elements and the reactions of the circuit being analyzed may be placed under the control of the contacts of the K_i relays.

*A convincing comparison in this respect is given in [4].

The windings of several auxiliary relays D_1, each with three windings, are also brought out to the plug-in board. These relays are used as supplementary relays, providing additional contacts should the number of C_1 relay contacts prove insufficient to model a specific surface undergoing analysis, or they are used for other special purposes. Similarly, auxiliary resistors, rectifiers, etc., are also provided. Direct current is supplied to the machine by means of several supply sockets that are provided both in the upper and lower parts of the plug-in board.

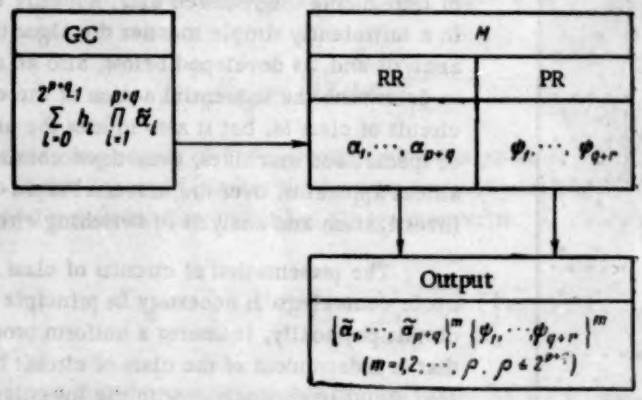

Fig. 3. Block diagram of the machine used for structural circuit analysis.

Electrical connections required in the model are set up manually on the plug-in board of the machine by the use of double-ended connecting cords, with a jack at each end, the jacks being plugged into sockets as required, in order to construct a model of the circuit being analyzed. An example of a model circuit, set up on the plug-in board of the machine is shown in Fig. 2.

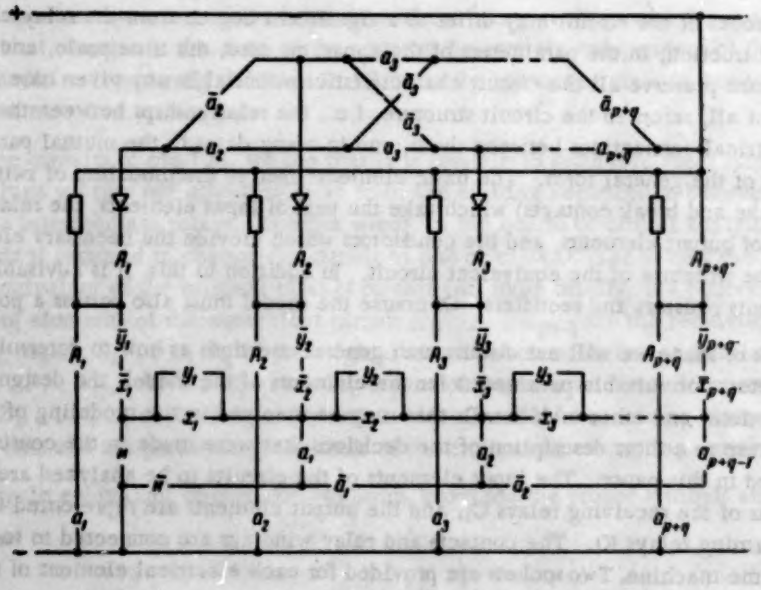

Fig. 4.

The realization in the machine of the algorithm of the structural analysis of the circuit is accomplished in the following manner. A special device, the constituent generator GC (Fig. 3), is used to feed all possible input combinations, in time sequence, to the receiving relay RR shown in the block diagram of model M (consequently also designating input elements to the equivalent circuit. When a circuit is being

analyzed to determine the required operating (nonoperating) conditions, should a certain output combination resulting from the action of the receiving relays of the model lead to the operation (nonoperation) of even one of the performing relays, PR, then the machine will stop and present the particular result of the analysis, which includes the input combination $\{\tilde{\alpha}_1, \ldots, \tilde{\alpha}_{p+q}\}$ and the corresponding output combination $\{\Psi_1, \ldots, \Psi_{q+r}\}$ of circuit elements. If the results are to be copied down manually they are obtained by means of lamp signals, after which the operator allows the machine to proceed. If the printing device is used, the machine proceeds with the problem automatically after it has finished printing the partial result.

The complete results of the analysis are presented in a table, the lines of which consist of a pair of combinations:

$$\{\tilde{\alpha}_1, \ldots, \tilde{\alpha}_{p+q}\}^m, \qquad \{\Psi_1, \ldots, \Psi_{q+r}\}^m. \tag{8}$$

Here $m = 1, \ldots, \rho$ denotes the number of the result in the sequence and ρ denotes the number of partial results of the analysis.

The method of analyzing a switching circuit by separating it into its constituent units possesses many advantages. To use the results of the analysis we simply compare them with the results obtained by synthesis as shown in truth tables for circuits of class O, or in action tables for circuits of class M. This type of analysis always results in a definite answer as to the relative merits of two circuits under comparison; allows us to determine the influence upon the state of several elements, of the action of very fast, short-time duration chains occurring due to a change in one of the cycles of a class M circuit; simplifies the uncovering of previously unforeseen work cycles that may occur during circuit operation, and so on.

However, this method possesses a major disadvantage, in that the length of the analytic process is doubled for each additional input element that is added to the circuit undergoing analysis.* This defect cannot be removed by constructing the machine entirely of elements that do not possess inertia because if we do not use a physical model of the circuit in our analysis, then a sharp increase occurs in the number of operations required to determine the number of closed paths that may exist in the circuit, not to mention the the difficulties involved in this case, in coding the circuit for presentation to the machine. However, this defect can be compensated for to a significant degree by building the circuit of the constituent generator in such a manner that n_1 of the input elements are kept constant for a series of inputs during the process of analyzing the input combinations. In this way the time interval that is required for the introduction of the various possible combinations is shortened 2^{n_1} times. We have done this in our machine by the use of the three-position switches XY, which are shown in the circuit diagram of the constituent generator (Fig. 4). With the switch in the upper position (x contacts closed) the paths which control the action of the corresponding receiving relays of the block diagram M are closed. With the switch in the lower position the circuit connections which control the receiving relays are open. Contacts $\underline{u}$ and $\overline{u}$ belong to the impulse generator relay; they control the operation of the constituent generator. The design of the constituent generator circuit is obtained analytically; this circuit insures a sequential presentation of the constituents in which each succeeding combination differs from the preceding combination by a change in only one factor.

3. Mechanization of the Sequential Operation of the Circuit Elements

The determination of the sequential action of the circuit elements is the most important stage in the analysis of the operation of class M switching circuits.

Now, the operation of the switching circuit is, in the end, determined by its structure. A complete understanding of the structure may be obtained by an analysis that is based upon the subdivision of the single-cycle equivalent circuit into its constituent units, and the results of this analysis may then be used to determine the sequential operation of the circuit elements. The solution of the problem consists in the construction of an algorithm that will enable us to transform the results obtained from the structural analysis (8), as listed in the form of a table of combinations, into a table that will give the required sequential information.

It is not difficult to show that such an algorithm may be built upon the basis of the obvious fact that for the single-cycle equivalent circuit the following relations hold:

*This follows from the fact that the number of constituent units is equal to 2^n, where $\underline{n}$ is the number of input elements in the circuit.

$$\{\tilde{a}_1,\ldots,\tilde{a}_{p+q}\}^m = \{\tilde{a}_1,\ldots,\tilde{a}_p,\ \tilde{x}_1,\ldots,\tilde{x}_q\}_t, \tag{9}$$

$$\{Y_1,\ldots,Y_{q+r}\}^m = \{X_1,\ldots,X_q,\ Z_1,\ldots,Z_r\}_{t+\tau} \tag{10}$$

and

$$\{\tilde{x}_1,\ldots,\tilde{x}_q\}_t = \{X_1,\ldots,X_q\}_t, \tag{11}$$

where $\underline{t}$ and $t+\tau$ are numbers designating succeeding steps in the resulting sequence table. Therefore, given the input combinations initially, and knowing the order of operation of the inputs upon the receiving elements, it is possible, with the aid of the algorithm, to uniquely determine the possible sequential operation of a class M switching circuit. Naturally, the sequence table that is obtained by this process is realizable.*

In spite of the high degree of formalization connected with the indicated algorithm, its realization is connected with significant difficulties due mainly to the fact that a memory with an enormous capacity is required to store the table (8). We were forced by the latter circumstance to seek a more rational method of solving the problem.

Let us assume that the circuit of class M that is to be analyzed is represented by a physical model of the corresponding one-cycle equivalent circuit. Let us also assume that the initial combination of input elements, as well as the program of external influences upon the receiving elements of the circuits, have also been provided. In order to represent the principal logical operation of the circuit in the machine by means of an algorithm, which is to be used to determine the sequential action of the circuit elements, we will introduce the following designations: M to represent the physical model of the single-cycle equivalent circuit, B to designate the circuit block of the machine that represents the initial combination of input elements and the changes in the settings of the receiving elements in the steady state of the circuit being analyzed, O to designate the block representing the return paths, which insure the transfer of the settings to the performing relays of the model, the latter relays functioning as the reacting organs of the intermediate elements of the circuit, corresponding to Eq. (11), D to represent the block that provides a time delay to take into account the operating or release times of the circuit elements, "Out" to represent the output of the machine which gives the settings of the circuit elements as time steps in a sequence table, $Y_A(t)$ to denote information obtained from block A at time $\underline{t}$, $G(Y_A(t) \to C)$ to represent the transfer of the information $Y_A(t)$ to block C, $C[Y_A(t) \sim Y_C(t)]$ to represent the operation of block C that results from the information $Y_A(t)$, changing the state of the letter to $Y_C(t)$, $p(A,C)$ to represent the logical conditions that determine the block (A or C) to which it is necessary to forward instructions, ω to represent the logical conditions that are always false.

Using the accepted conditions (11), we obtain the logical scheme for the algorithm in the following form:

$$\begin{aligned} &\overset{1}{\downarrow} BG(Y_B(t) \to \text{Out}\quad M) M(Y_B(t) \sim Y_M(t)) \overset{2}{\downarrow} G(Y_M(t) \to 3) \\ &D(Y_M(t) \sim Y_D(t)) G(Y_D(t) \to \text{Out}, O) O(Y_D(t) \sim Y_O(t)) \\ &G(Y_O(t) \to M, B)\, p(M,B) \overset{1}{\uparrow} M(Y_O(t) \sim Y_M(t))\, \omega \overset{2}{\uparrow}. \end{aligned} \tag{12}$$

The block diagram of the machine, shown in Fig. 5, is based upon the principles developed in the logical analysis (12). The machine operates in accordance with the principles outlined as follows.

The first combination of input elements for the circuit being analyzed

$$\{\tilde{a}_1,\ldots,\tilde{a}_p\}_0 + \{\tilde{x}_1,\ldots,\tilde{x}_q\}_0, \tag{13}$$

is placed into the machine with the three-positional switches** of the input block B in the upper position, and

*The table of sequential actions for class M circuits is realizable, if and only if, it does not contain at least two steps $\underline{t}$ and $t+\Delta t$, $\Delta t > 1$, with the same combinations of receiving and intermediate elements, each of which is immediately followed by steps requiring different combinations of the intermediate and performing elements.

**These are the same three-positional switches XY that are shown in the block diagram of the constituent generator (Fig. 4).

transferred at the same time both to the receiving relay RR of the physical circuit model M and to the output of the machine, where it is placed in the zero time position in the output table of results. The input combination (13) leads to the output combination

$$\{X_1, \ldots, X_q\}_\tau + \{Z_1, \ldots, Z_r\}_\tau, \tag{14}$$

which is obtained from the performing relay PR of the physical circuit model M, this output combination then enters the time-delay block D. The latter operates upon the combination (14) in such a manner as to prevent any change in the elements of the above combination (prevents the operation or release of any of the delayed elements) during the time interval $s\tau$ of the delay. After leaving the time-delay block D the output combination together with the combination

$$\{\tilde{a}_1, \ldots, \tilde{a}_p\}_t \tag{15}$$

of the remaining receiving elements, which have not undergone any change, is fed into the output of the machine as the first line in the output table. At the same time, the combination of intermediate elements

$$\{X_1, \ldots, X_q\}_t \tag{16}$$

enters the feedback block O which feeds the combination to the corresponding receiving relays of the model M. In this manner, if the circuit has not reached a stable condition, its receiving elements are fed a new combination that differs from the combination (13) and the process repeats itself.

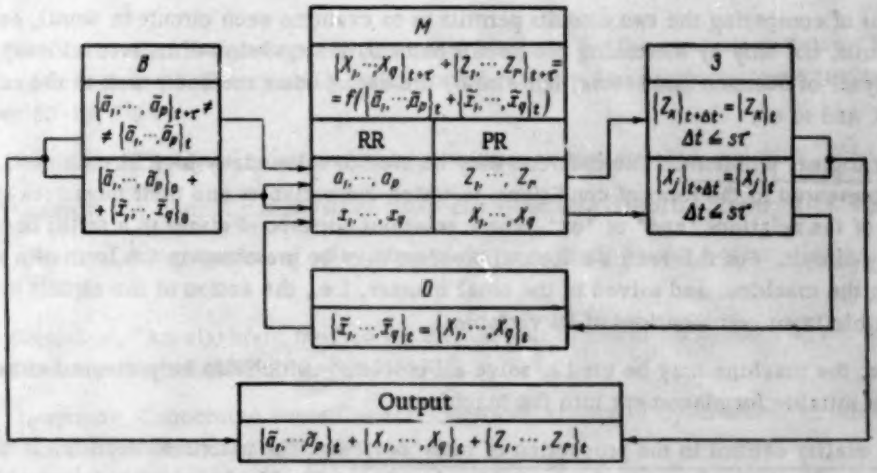

Fig. 5. Block diagram of a machine used to determine the sequential operation of the elements of a circuit.

The stable condition of the circuit is characterized by the equations

$$\{X_1, \ldots, X_q\}_{t+\tau} = \{X_1, \ldots, X_q\}_t, \tag{17}$$

$$\{Z_1, \ldots, Z_r\}_{t+\tau} = \{Z_1, \ldots, Z_r\}_t, \tag{18}$$

whose action is controlled by a special loop of the machine (not shown in the block diagram of Fig. 5) and corresponds to an untrue result of the logical conditions p(M,B) of (12). When conditions (17) and (18) occur, the machine stops and, in order that the process continue, it is necessary in the general case, to change the setting of the receiving elements of the circuit. This is accomplished when the operator sets the switch of input block B to the lower position.

The moment during which the machine completes the process of obtaining the table of output values is not

automatically determined because this would require that the machine be provided with a memory of unjustifiably large capacity. Practically speaking, the operator can easily determine this moment since, as a rule, it coincides with the stable state of the circuit which occurs at the same time that the machine stops.

In order to facilitate manual recording of the results, the table of output values is placed, in parts, on an illuminated board. The machine may also be constructed, by means of an additional attachment, so that it prints the steps of the resulting table of combinations automatically.

4. Other Applications of the Machine

Analysis of switching circuits constructed of solid-state elements. The possibility of using a machine that has been constructed to analyze relay switching circuits for the analysis of circuits constructed of solid-state elements is based upon the fact that any such circuit may be represented by an equivalent relay circuit, which may be placed into the machine and investigated in the usual way.

The machine is constructed in such a manner that it is possible, for example in an electron-tube circuit, to determine the operating conditions at any point of the circuit for all possible combinations of the input conditions, the operation of the machine being limited only by the action of control voltages applied at the outputs of the circuit.

Comparison of relay circuits. The two circuits that are to be compared are placed into the machine in such a manner that they make use of the same performing relays of the machine; however, a separate winding is used for each circuit, the windings being placed on each relay, series-aiding. Therefore, during the structural analysis, the machine will stop only for those constituent units of the circuits being analyzed which are present in one circuit and absent in the other.

The process of comparing the two circuits permits us to examine each circuit in detail, and enables us to simplify the circuits, not only by shortening the circuit paths by the omission of individual relay contacts, as is done by the analyzer of Shannon and Moore, but also by the use of other methods, such as the combination of similar elements, and so on.

Solution of logical problems. The machine may be used to solve many logical problems, since any such problem can be presented in the form of conditions, in which the variables and their negatives are bound together by means of the relations "and" or "or." These relations correspond either to a series or to a parallel connection of a relay circuit. For this reason a logical problem may be presented in the form of a relay-contact circuit, placed in the machine, and solved in the usual manner, i.e., the action of the circuit may be investigated for all possible input combinations of its variables.

In principle, the machine may be used to solve all problems, which can be presented either directly or indirectly in a form suitable for placement into the machine.

Automatic quality control in the production of relay devices. The machine described in this paper may be used to automatically detect errors in the assembly of various relay blocks, or other irreparable circuit errors occurring during the manufacturing process. We wish to point out that the first models of the machine were not designed with this purpose in mind, and for this reason they possess a series of limitations in this respect (such as the presence of only one source of supply, the absence of automatic means of entering the operating conditions of the circuits under test into the machine, etc.). However, the principles upon which the operation of the machine is based permit it to be used successfully to control the quality of series-produced electrical apparatus and devices, if various specialized trial and test stations are established in the production line.

SUMMARY

Experience gained during the use of the machine described in this article for the solution of the practical problems involved in the analysis and design of relay-switching circuits confirms its effectiveness. We have found that the use of the machine is a great advantage in the analysis of multi-element, multi-step circuits.

The machine described in this article, which is used to analyze the operation of relay circuits, received high praise in the foreign technical press, and from among the other automatic devices it was chosen for the Grand Prize at the exposition held in Brussels in 1958.

At the present time we find ourselves at the stage of creating universal machines that are capable of designing relay circuits. Machine-circuit synthesis, a process of extremely great value, will undoubtedly require machine analysis as the final step of circuit design. In this case, the machine that is used for the circuit analysis may be substantially simplified if the data that represent the circuit to be analyzed are automatically introduced into the machine, since this will permit us to remove the circuit loops that are required in connection with the manual control of the machine occurring during the process of obtaining and verifying the output table. The most likely way of introducing the circuit to be analyzed into the machine seems to be through the use of punched cards.

The author wishes to thank Prof. M.A. Gavrilova for the valuable observations and advice which she gave him while he was working on the design of the machine, which was to be used for the analysis of relay circuits.

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ELECTRONIC PROGRAM-CONTROLLED COMPUTERS

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Digital methods have to be used in solving certain automatic control problems (e.g., those considered in [1]).

In some cases it is best not to use digital computers however, because the computer is complex, and digitalizing devices are required. Also, it is sometimes necessary to carry out many operations in order to solve a relatively simple problem. Combinations of digital and analog methods are therefore advantageous [2].

General-purpose and specialized analog computers have many units in common. For example, the MN-8 has twelve multiplying units, about 40 nonlinear units, many operations amplifiers, etc. The desired type of machine should include a high-grade specimen of each of these types of unit.

Switching (program) units enable one to use these units as required to solve equations.

These computers differ from analog computers, although they have much in common with them, because they include digital devices.

It is convenient to call computers of this type electronic program-controlled computers (EPC for short).

They differ from analog machines in that they have all the features of digital machines, have many less operations amplifiers, and can be programmed to solve several distinct problems automatically, i.e., they can control several distinct processes at once.

They can solve problems in logic, can select the optimum solution from many alternatives, and can solve problems that cannot be handled by analogs (e.g., can find the solution to a convolution integral).

The design principles of such computers are presented here.

1. EPC Working Principles

Figure 1a illustrates the working principles of this type of machine.

The computer is connected by the switches $k_1, k_2, \ldots, k_n, k'_1, k'_2, \ldots, k'_n$ to solve the problem during certain time intervals t_1. The program unit supplies to the computer, in the appropriate sequence voltage pulses whose amplitudes are proportional to the inputs $U_{i_1}, U_{i_2}, U_{i_3}, \ldots, U_{i_n}$. The pulses produced by the computer are suitably manipulated and are distributed, via the various channels, to the output units $BY_1, BY_2, \ldots, BY_n$.

The voltages reaching these units are stored, and are used in subsequent calculations or to control some process.

Figure 1b shows (at 0) the pulses supplied by the time-interval generator TIG, which pulses control the program unit. Diagrams 1, 2, 3, . . . , n show the outputs from the output units. The actual law followed by the function is shown by the broken line. Diagram Σ shows the output from the computer unit.

The principle is the pulse-height principle often used in time-division telemetry. The system differs from telemetry systems in that:

a) the computer unit replaces the communication channel;

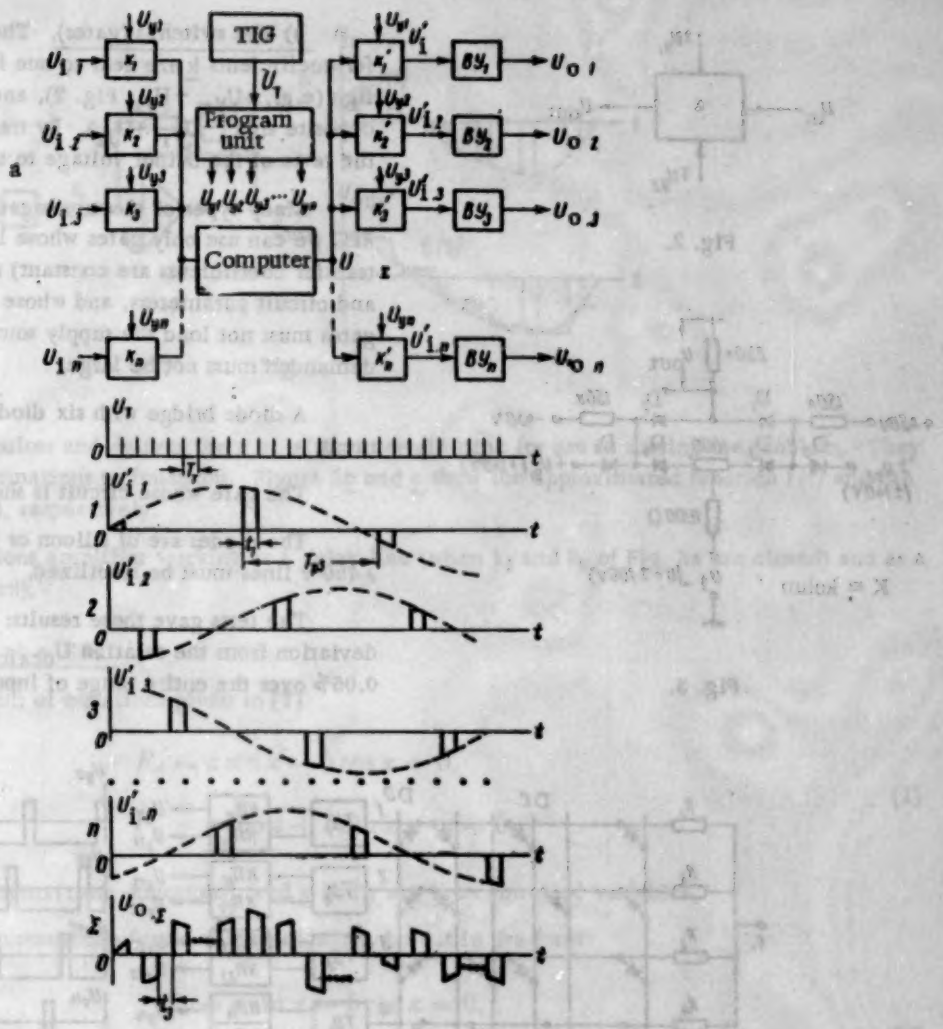

Fig. 1.

b) the transducers, receivers and communication channel (computer unit) are combined into one;

c) the transfer coefficient of the system is determined by the transfer coefficient of the computer unit, whereas this coefficient must be constant if a communication channel is used, and so on [3, 4].

2. EPC Units

Analog computer units are always present; their numbers are determined by the operations to be performed.

These units include functional converters, differentiators, integrators, and so on. They are described in detail in a number of places, and so we shall not dwell upon them.

We consider only certain special requirements on units used in EPC computers, and some particular aspects of the program-control units.

a) The computing devices. Diagram Σ of Fig. 1 shows that these devices work as sampled-data units. However, any electronic system has a finite passband. Therefore, the frequency range of the controlled object is restricted by the fact that these devices are used repeatedly for different purposes. This frequency range is taken at present, so the passband and the pulse parameters must be chosen to correspond.

The passband of such a device is controlled by the bandwidth of the wide-band operations amplifiers (OA). These amplifiers must be drift-free, to ensure stable working. The amplifiers designed at the Institute of Automation and Remote Control [5] are adequate in this respect. The passband in the direct negative feedback mode is 10-15 kc for a signal of maximum amplitude 100 v.

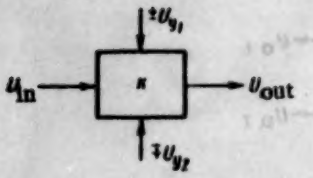

Fig. 2.

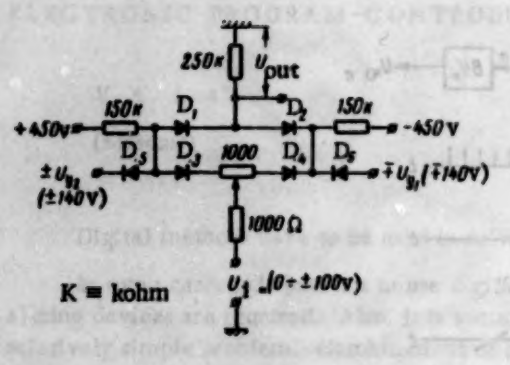

Fig. 3.

b) The switches (gates). These are devices whose transfer coefficients k are near to one for control voltages of one sign (e.g., $+U_{y_1}$, $-U_{y_2}$, Fig. 2), and zero for voltages of the opposite sign ($-U_{y_1}$, $+U_{y_2}$). By transfer coefficient we mean the ratio of the output voltage to the input voltage.

Many types of electronic gates have been described. In EPC we can use only gates whose linearity is good (whose transfer coefficients are constant) and independent of voltage and circuit parameters, and whose passbands are wide. The gates must not load the supply sources, and the control voltages demanded must not be large.

A diode bridge with six diodes [6] satisfies this requirement.

The gate whose circuit is shown in Fig. 3 has been tested.

The diodes are of silicon or hard-tube types. The ±450-v lines must be stabilized.

The tests gave these results: transfer coefficient 0.9265; deviation from the relation $U_{ocalc} = 0.9265\ U_i$ not more than 0.05% over the entire range of inputs; passband up to 20 kc.

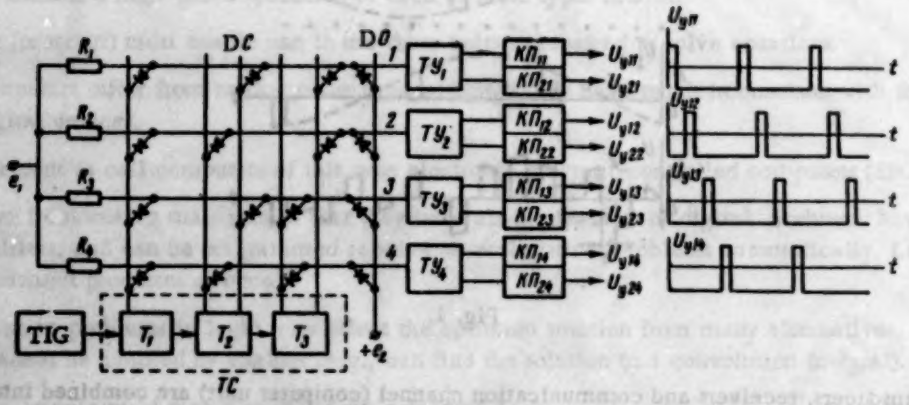

Fig. 4.

c) Program unit. This must supply the control voltages to the gates in the appropriate sequence. The output voltages must be of appropriate magnitudes, and of both signs. The stages must have power sufficient to control several gates.

A simple program unit is shown in Fig. 4; it uses the above diode gate.

This unit contains a time-interval generator TIG, trigger circuits TC, a diode network DC, diode limiters DO, control triggers TY and cathode followers KП. The triggers are such as to provide plate voltage changes of ±140 v.

d) The output units. These units BY receive pulses of large mark-to-space ratio:

$$Q = T_p/t_1 = 2NT/T = 2N,$$

where T is the width of a control pulse, T_p is the time needed to solve the problem ($1/T_p = f_p$ = frequency with which the problem is solved) and N is the number of times the computer unit is used when the problem is solved once.

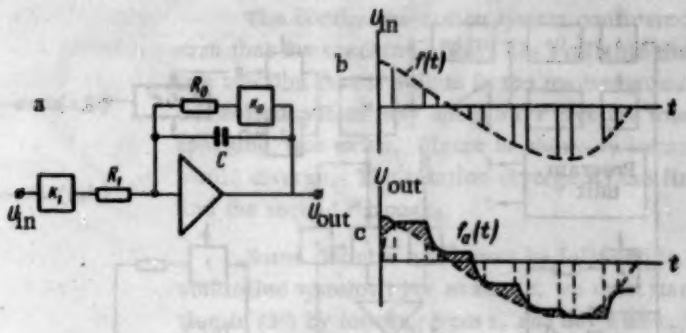

Fig. 5.

These units store the pulses and deliver them up after a certain time for use in solving the problem. They also provide stepwise approximations to functions. Figure 5b and c show the approximated function f(t) and the approximating function $f_a(t)$, respectively.

Each unit is an operations amplifier working as a delay line (when k_1 and k_0 of Fig. 5a are closed) and as a store (when k_1 and k_0 are open).

3. Block Diagram of the EPC

Consider how the system of equations given in [7]

$$F_x = a\sin x - b\cos x = 0,$$
$$F_y = a\cos x + b\sin x - y = 0, \tag{1}$$

is solved; here a and b are inputs (time-functions), and x and y are the controlled variables.

System (1) may, for convenience in use with computers, be put in the form

$$F_x = a\sin x - b\cos x = 0,$$
$$F_y = y - \sqrt{a^2 + b^2} = 0. \tag{1'}$$

We do not need to follow the solution right through to the numerical results in order to illustrate the principles. No special symbols are needed to denote the scales in which the machine operates.

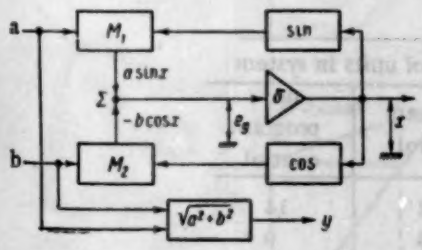

Fig. 6.

Figure 6 shows the block diagram of an electronic device to solve system (1').

The symbols are: M_1 and M_2, multipliers; sin and cos, corresponding functional converters; δ, an amplifier of high gain, or a difference amplifier; and Σ, a summation point.

The device must be checked for steady-state stability when the circuit is compiled [7].

Figure 7 shows a circuit to solve (1') by digital methods.

The first equation of (1) is solved in two stages.

Stage 1. The gates denoted by 1 are opened by the control voltages. The product, $a\sin x_1$, is fed to BY_1, where it is stored and inverted.

Stage 2. The gates denoted by 2 are opened. The product, $b\cos x_2$, is fed to the difference amplifier δ. The stored product, $a\sin x_1$, is also fed to this amplifier.

The output x_2 from δ varies in such a way that the input, $a\sin x_1 + b\cos x_2$, is almost zero. If $a\sin x_1 + b\cos x_2 = 0$, then x_2 is the first approximation to the solution of (1'). The root to the first equation in (1') is obtained by iteration.

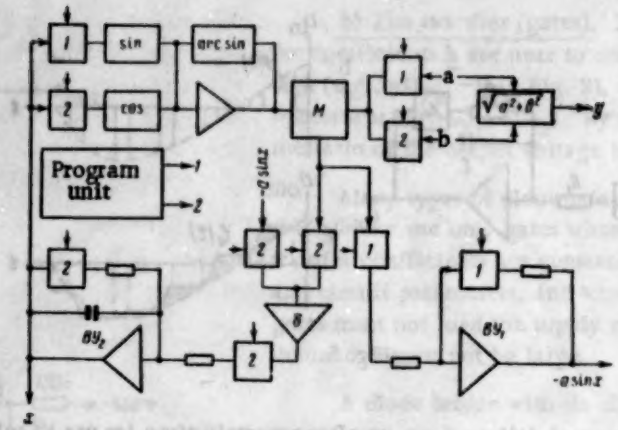

Fig. 7.

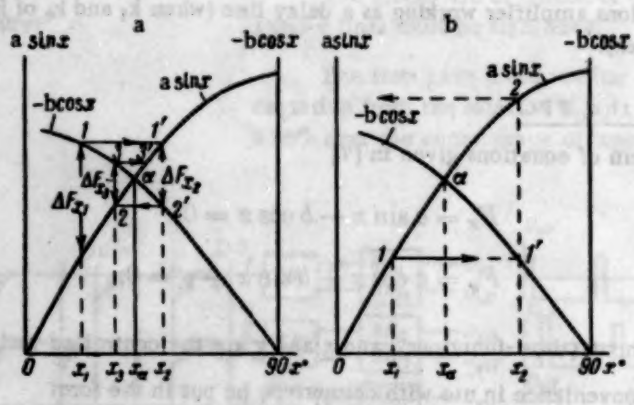

Fig. 8.

Figure 8a gives a graphical interpretation of the way in which the first equation of (1') is solved.

Let the device be in a state such that the output from δ is x_1, and let $b\cos x_1$ be produced in the first stage. In the second stage, $a\sin x_2$ is compared with the voltage produced previously. The output from the amplifier is then a voltage x_2 such that $a\sin x_2 - b\cos x_1$ is zero. This completes the first approximation. The process is then repeated. Several repetitions give us a voltage x_α that is the solution to the first equation of (1'). The second equation of (1') is solved as a result of the solution of the first.

No. of systems of type (1)	Unit	No. of units in system	
		program control	no program control
1	OA	12	14
	Functional converters	6	9
	Gates	7	–
	Total diodes (roughly)	147	165
2	OA	15	28
	Functional converters	6	18
	Gates	20	–
	Total diodes (roughly)	225	330
3	OA	18	56
	Functional converters	6	36
	Gates	36	–
	Total diodes (roughly)	320	660

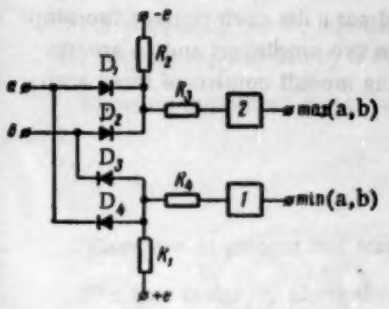

Fig. 9.

The continuous-action system considered previously can be made such that the machine always has available the solution, provided there is a solution that is unique in the mathematical sense. The program-controlled system may not always give the solution, although, formally speaking, one exists. Figure 8a shows an instance in which the solution would diverge. The solution diverges if the first stage produces $a \sin x_1$ and the second $\rightarrow b \cos x_2$.

Some definite rules must be followed in designing such program-controlled systems. For example, we must start to solve the first equation in (1') by forming $b \cos x$, and not $a \sin x$.

Thus, the solution to (1') will converge only if the factors a and b are supplied in some suitable sequence. The lesser (in absolute value) must be the first to be supplied.

A diode circuit (Fig. 9) may be used to separate the minimum and maximum factors. Of the pair D_1 and D_2, the one that receives the larger voltage will conduct; and of D_3 and D_4, the one that receives the lesser voltage [8].

It is possible to transform the equations in such a way that the number of iterations is minimized.

Consider a graphical interpretation of one suitable method, as applied to the first equation of (1'). We transform the equation to

$$x = x - a \sin x + b \cos x.$$

To solve this on the EPC we put it in the form

$$x_{i+1} + a \sin x_{i+1} = x^i + b \cos x_i,$$

where i is the number of the iteration.

Figure 10 shows that the rule given above must be obeyed in solving the transformed equation.

In one instance, where a system of the type of (1') had to be solved, the program method required far less computer units than did a normal analog method.

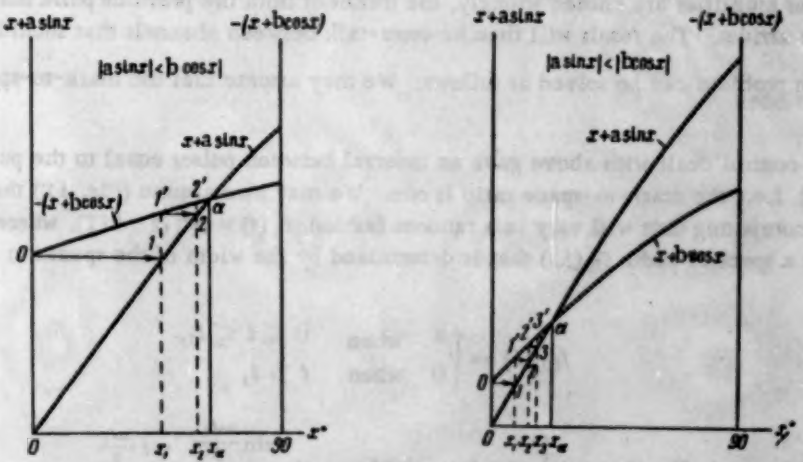

Fig. 10.

The table given below illustrates the gain to be obtained from program control when one, two or three systems of the type of (1') are to be solved. Here we have assumed that each multiplier has three operations amp-

lifiers and two squaring units, each of which contains 15 diodes, that the sin and cos units each contain two amplifiers and an arc-sin converter (30 diodes), that the sin-to-cos converter contains two amplifiers and an arc-sin converter, that the gates each contain six diodes, and that the device for deriving moduli consists of three amplifiers and three squaring units.

4. Some Technical Features of the EPC

The following aspects require attention in designing such systems:

a) the required repetition frequency (time) must be decided on;

b) the signal-selection system at the input requires consideration;

c) the necessary passband in the computer units must be calculated or, if this passband is fixed, the frequency range of the controlled object must be established.

If we assume that the problem is solved in a single iteration, the period needed for a single solution is determined by the following factors:

a) the method of approximating the functions;

b) the maximum permissible error in the approximation (ΔF_a), and

c) the way in which the functions and their derivatives vary.

The error in the approximation is given by

$$\Delta F(t) = F_a(t) - f(t) \leq \Delta F_a,$$

where $F(t)$ is the function to be approximated and $F_a(t)$ is the output produced by the output unit.

Piecewise-linear and step-function approximations can be used. The first is the more accurate, but the function and its first derivative have both to be stored. The second is less accurate, but only the function has to be stored.

If the second method is used, the time needed to solve the problem is

$$T_p = \Delta F_a / F'(t)_{max}.$$

The method of selecting the signals has to be considered, because, if the parameters of the program unit and of the operations amplifier are chosen wrongly, the transient from the previous pulse may not have died away when the next pulse arrives. The result will then be cross-talk between channels that should be separate.

This selection problem can be solved as follows. We may assume that the mark-to-space ratio of the pulses is constant.

The program control dealt with above gave an interval between pulses equal to the pulse length, t_1 (t_2 is the unused interval), i.e., the mark-to-space ratio is one. We may also assume (Fig. 1Σ) that the pulse height ξ_i at the input to the computing unit will vary in a random fashion, $F_i(t) = \xi_i f_1(t - 2iT)$, where $i = 1, 2, \ldots, N$. The signal then has a spectral width $G_1(j\omega)$ that is determined by the width of the spectrum from a single pulse [9]:

$$f_{in}(t) = \begin{cases} \alpha & \text{when} \quad 0 \leqslant t \leqslant t_1, \\ 0 & \text{when} \quad t > t_1 \end{cases}$$

$$G_{in}(j\omega) = \int_{-\infty}^{\infty} f_{in}(t) e^{-j\omega t} dt = \alpha t_1 \frac{\sin \frac{\omega t_1}{2}}{\frac{\omega t_1}{2}} e^{-j\frac{\omega t_1}{2}}.$$

The amplitude-phase response of the computer is $Y(j\omega)$, so the output signal is a time-function

$$f_{out}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{in}(j\omega) Y(j\omega) e^{j\omega t} d\omega.$$

For simplicity we assume that the phase characteristic is zero, and that the frequency-amplitude characteristic may be represented by a step function; the expression can then be calculated from the series expansion.

Knowing the output signal, we can make t_3 such as to reduce the cross-talk between channels to a suitable level.

SUMMARY

There are at present two ways in which analog computers are developing.

The first is simply along the lines of electromechanical analogs.

The second is new, and involves structural systems in which the computing units are used repeatedly to solve many equations with similar structures. Machines of this type are called electronic program-controlled analogs.

This new type of machine economizes on computing units, and has the properties of a digital system; it can solve a far wider range of problems.

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PHOTORESISTORS AS CIRCUIT ELEMENTS

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The main characteristics of commercial photoresistors are considered in relation to circuit uses in terms of the general laws of the internal photoelectric effect. It is shown that the best basic parameters to use are $A_0 = e\mu_0 k\beta_0\tau_0/d^2$, and the coefficients γ and α, which, respectively, define the nonlinearity of the voltage-current and intensity-current characteristics and the permissible power dissipation, and the mean carrier lifetime τ_0. Methods of calculating the load and frequency responses, and temperature compensation circuits, are given.

The equations enable one to calculate photoresistor circuits with an accuracy that is adequate for practical purposes.

Developments in semiconductor technology in the last 5-10 years have made photoresistors normal elements in many signalling, switching, counting and automatic control circuits [1-16].

Photoresistors have much higher values of integral sensitivity and of permissible photocurrent than vacuum photocells do; they are also smaller. Their designs are simpler, and do not involve evacuated tubes. They work stably, and show no appreciable aging or drift. The average service lives of the usual commercial photoresistors are measured in tens of thousands of hours.

The main disadvantages are the relatively slow response, the relation of response time to light intensity, the sensitivity of the dark current and the photocurrent to temperature, and the absence of saturation regions on the voltage-current curves, which latter makes the sensitivity dependent on the voltage applied.

Table 1 and Fig. 1 give the basic parameters of commercial photoresistors. Data are also given in Table 1 for photodiodes and small photomultipliers, for comparison.

These data show that existing types are sensitive to the near ultraviolet, to visible light, and to near infrared radiation. Some types (FSK-M, FSS-M) are highly selective, others have broad sensitivity regions (FS-K, FS-D, FS-A, FS-B). Some [e.g., FSK-M, FSS-M, FS-K (copper doped)] are very sensitive to α-, β-, γ- and x-rays [1-4].

Table 1 also shows that photoresistors based on cadmium compounds have the largest integral sensitivity to visible light (e.g., FSK-M, FSS-M, FS-K, FS-D). These types can be made to have areas ranging from less than a mm^2 to several cm^2, and to carry currents ranging from microamps to amps.

Photoresistors have several features that influence calculations on circuits in which they are used. These features have their source in the laws of the internal photoelectric effect.

These laws are determined by the structure of the semiconductor, by any impurities present, by the nature of the optical transitions, and by the type of contact used. In general, the photocurrent I_p is related to the light intensity L by

$$I_p = \frac{C_1 L^{\alpha^*} + C_2 L^{\beta^*}}{(B_1 + L)^{\gamma^*}}, \qquad (1)$$

TABLE 1

Characteristics of Some Photoresistors

Parameter	Photoresistor									Photodiode	Photomultiplier
	type										
	FS-A1	FS-B2	FS-K0	FS-K1	FS-K2	FSK-M1	FSK-M2	FSS-M****	FS-D	FD-1	FEU-25
material			polycrystalline CdS			monocrystalline CdS		mono-crystalline CdSe	poly-crystal-line CdSe	mono-crystal-line Ge	–
Working area, mm	3×7	11×11	4×7,2	4×7.2	4×7.2	1×2	1.1×1.5	1×2	4×8	1×1	500*****
Working voltage, v	15—30	30—60	100—300	100—300	100—300	60—150	60—150	10—100	100—200	100—150	1000
Mean sensitivity under best conditions, amp/lumen	0.02	0.012	1.2	1.2	1.2	2	2	4—10	12	0.03	1—28
Dark resistance, ohms	10^4—10^5	10^5—10^7	$>10^6$	$>10^7$	$>10^6$	$>10^{10}$	$>10^8$	$>10^8$	$\geqslant 10^5$	$<10^6$	$<5\cdot10^{-3}$
Wavelength of maximum sensitivity, mμ	2.1	0,7	0.52	0.60	0.52	0.51	0,52	0.74	0.78	1.5—1.6	0.39
Temperature coefficient of photocurrent, $°C^{-1}$	$1.6\cdot10^{-2}$***	10^{-2}	$1.2\cdot10^{-2}$***	$1.4\cdot10^{-2}$***	$1.2\cdot10^{-2}$***	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-1}	—
Mean photocurrent decay time, sec*	10^{-5}	10^{-2}	10^{-1}—10^2	10^{-2}	10^{-1}—10^{-2}	10^{-2}—10^{-3}	10^{-2}	10^{-3}—10^{-4}	10^{-3}	10^{-5}	10^{-9}

*Decay approximated by an exponential.

**Over the range 0-40°C.

***Over the range ±100°C.

****Experimental.

*****Area of photocathode, mm^2.

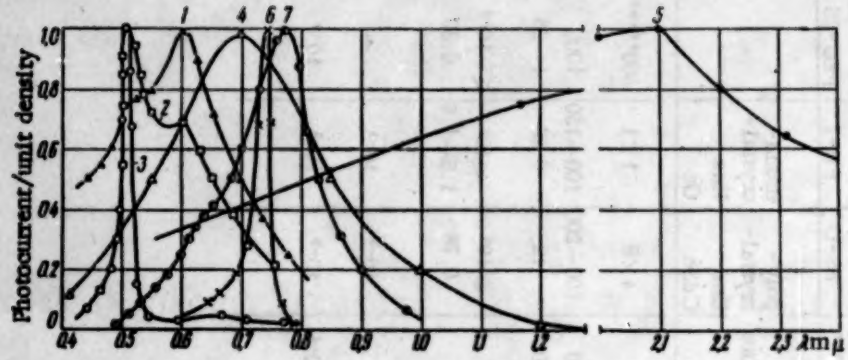

Fig. 1. Spectral sensitivity curves for: 1) FS-K1; 2) FS-K2; 3) FSK-M; 4) FS-B2; 5) FS-A1; 6) FSS-M; 7) FS-D.

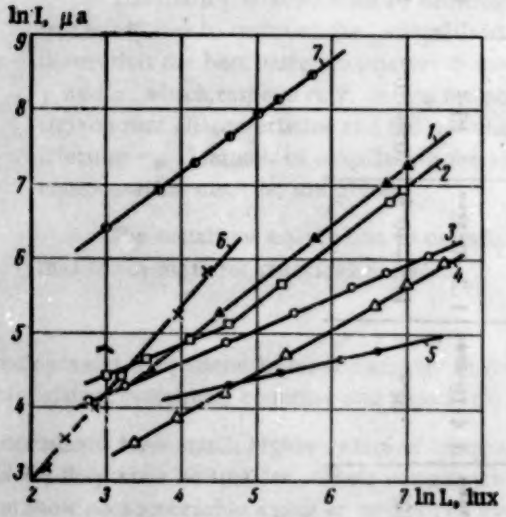

Fig. 2. Typical intensity-current curves for photoresistors: 1) FS-K1, U = 50 v; 2) FS-K, U = 50 v; 3) FS-B2, U = 40 v; 4) FSK-M, U = 20 v; 5) FS-A1, U = 15 v; 6) FSS-M, U = 13 v; 7) FS-D, U = 50 v.

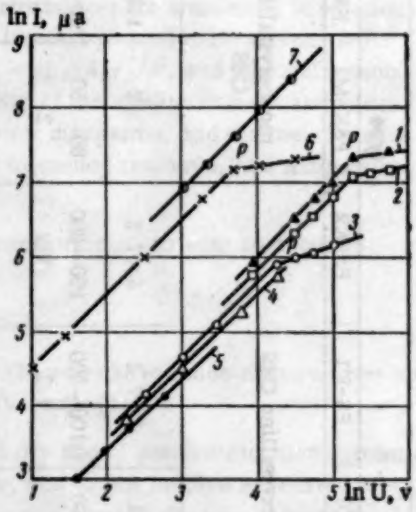

Fig. 3. Typical voltage-current curves for photoresistors: 1) FS-K1, L = 200 lux; 2) FS-K2, L = 200 lux; 3) FSK-M, L = 200 lux; 4) FS-B2, L = 200 lux; 5) FS-A1, L = 100 lux; 6) FSS-M, L = 10 lux; 7) FS-D, L = = 200 lux.

where C_1, C_2, B_1, α°, β°, γ° are constants for any given type of photoresistor.

The experimental curves of Fig. 2 fit (1) very well; these curves are typical ones for commercial photoresistors. Figure 2 shows that we can, for practical purposes, approximate a given intensity range by a power law

$$I_p = AL^{\alpha}, \tag{2}$$

where A and α are constants.

Now $I \propto \exp U$ for nonohmic contacts [5], and so we can [2] represent the voltage-current relation by a power law also:

$$I_p = BU^{\gamma}, \tag{3}$$

where U is the voltage applied, and B and γ are constants. If the contacts are ohmic, $\gamma = 1$. Figure 3 illustrates (3); it gives curves for the usual photoresistors.

The steady-state photocurrent in a semiconductor is [6, 7] related to L and U by

$$I_p = \frac{e\beta k\ \mu\tau}{d^2} LU_p, \tag{4}$$

where e is the electronic charge, k is the absorption coefficient, β is the quantum yield, τ and μ are, respectively, the mean lifetime and mobility of the carriers, and d is the distance between the contacts.

It has been shown [3, 6, 8] that, in general, $\tau = \tau(L)$, $\beta = \beta(L)$, $\mu = \mu(U)$. The mobility is related to the voltage [3] by

$$\mu = \mu_0 U^{\gamma'}, \tag{5}$$

where μ_0 depends on the semiconductor and γ' is a coefficient.

We may relate the lifetime and quantum yield to L in a first approximation by expanding the functions in series form, assuming that the constant term is zero. We then have

$$\tau = \tau_0 L^{\alpha'}, \quad \beta = \beta_0 L^{\alpha''}, \tag{6}$$

where α', α'', τ_0, β_0 are constants.

We substitute (5) and (6) into (4) to get

$$I_p = A_0 L^{\alpha} U^{\gamma}, \tag{7}$$

where

$$A_0 = ek\beta_0\tau_0\mu_0/d^2. \tag{8}$$

Now A_0, α and γ are to be considered as parameters of the photoresistor; they can be used to calculate all the quantities given in Table 2.

TABLE 2

Parameters of Some Photoresistors

Parameter	Type						
	FS-A1	FS-B2	FS-K1	FS-K2	FSK-M	FS-D	FSS-M
Power dissipation, mw	10	100	100	150	10	400	30
Voltage nonlinearity coefficient, γ	1	1	1	1	1*	1	1**
Intensity nonlinearity coefficient, α	0.4	0.5	0.8	0,5—0.8***	0.5	0.8	1.2
Approximate carrier lifetime τ_0, sec	10^{-5}	10^{-3}	10^{-3}	10^{-2}	10^{-3}	10^{-3}	10^{-4}
Temperature coefficient of total current, %/°C, for the range 20-100°C	—0.6	1.2	0.8	0.3	0.8	1.5	1.5

*Aluminum or cadmium contacts.

**Cadmium contacts.

***The lower value of α corresponds to lower intensities.

We see from comparing (7) with (2) and (3) that the coefficients of proportionality in the characteristics are, respectively,

$$A = A_0 U_p^{\gamma}, \quad B = A_0 L^{\alpha}. \tag{9}$$

It is found that A_θ is not strictly constant for photoresistors of any one type. The values of A_θ may differ by 50% or more. The causes are variations in electrode spacing and chemical composition, particularly the latter. Equation (7) shows that $A_\theta \propto 1/d^2$. Hence, A_θ changes greatly with small changes in d.

Types FS-K, FS-A and FS-D have relatively large values of d (3-5 mm), and the fluctuations are only 5 to 10%. Type FSK-M (monocrystals) has d ≈ 1 mm, and the electrode spacings may vary by 20-50%.

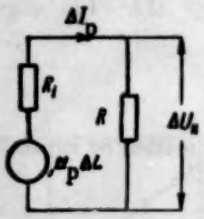

Fig. 4. Equivalent circuit of a photoresistor.

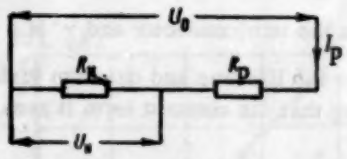

Fig. 5. Derivation of the optimum load resistance.

It is clear that d must be kept strictly constant if photoresistors are to be treated as interchangeable circuit elements.

Besides A_θ, we also have α and γ as parameters; these specify the sensitivity and resistance.

It is usual to give the static sensitivity, which is

$$K_p = \left(\frac{I_p}{L}\right)_{U_{p=\text{const}}}. \tag{10}$$

Now (7) and (10) show that

$$K = A_\theta L^{\alpha-1} U. \tag{11}$$

Hence, K is not really a parameter, because the nonlinearity causes it to have a different value at each point on the curve. The mean K may be used in calculations on the permissible voltage for a given intensity, in which the permissible power rating P_{max} is used. This power is found from the bend on the voltage-current curve (point P in Fig. 3). If $P > P_{max}$, the temperature rises rapidly, the photocurrent falls, and the unit is damaged. Now (7) and (11) show that

$$P_{p\max} = I_p U_{p\max} = A_0 L^{\alpha} U_{p\max}^{1+\gamma} = K_p L U_{p\max}, \tag{12}$$

so

$$U_{p\max} \frac{P_{p\max}}{K_p L}. \tag{13}$$

Consider a small increase in photocurrent in relation to $I_p = f(L,U)$; we have

$$\Delta I_p = \frac{1}{P_i}(\mu_p \Delta L + \Delta U_p), \tag{14}$$

where $\mu = R_i K_i$; K_i and R_i are the differential sensitivity and differential resistance, respectively, and are given by

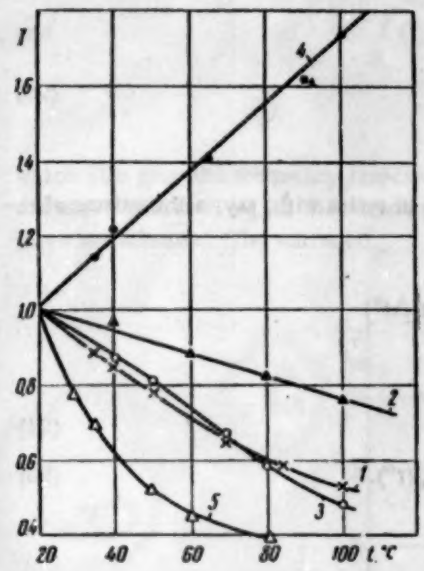

Fig. 6. Relative change of current with temperature. The total current at L = = 100 lux and U = U_H is taken as unity. 1) FS-K1; 2) FS-K2; 3) FSK-M; 4) FS-M1; 5) FSS-M.

$$K_i = \left(\frac{\Delta I_p}{\Delta L}\right)_{U_\phi = \text{const}},$$

$$R_i = \left(\frac{\Delta U_p}{\Delta I_p}\right)_{L = \text{const}}. \quad (15)$$

Now K_i and K are related by

$$K_i/K = \alpha, \quad (16)$$

which formula should be used to calculate K_i.

Figure 4 shows the equivalent circuit derived from (14). If R is the input resistance of an amplifier, the output parameter of the circuit in Fig. 4 is ΔU_H. Now

$$\Delta U = -\Delta U_H = -\Delta I_p R_H, \quad (17)$$

so, solving (14) and (17) for ΔU_H, we have

$$\Delta U_H = \mu_p \frac{\alpha_H}{1+\alpha_H} \Delta L, \quad (18)$$

where $\alpha_H = R_H/R_i$.

On the other hand, the load should receive the maximum power in the steady state. Figure 5 shows that

$$P_H = I_H U_H = \frac{R_H}{(R_H + R_s)^2} U_0^2, \quad (19)$$

where $R_s = U/I_p$ is the steady-state resistance of the cell. Now $dP_H/dR_H = 0$ occurs when $R_H = R_s$, so $\alpha_H = R_s/R_i$. Then (3) and (15) give us that

$$R_s/R_i = \gamma, \quad (20)$$

which formula may be used to calculate R_i. Substituting (20) into (18), we have

$$\Delta U_H = \mu_p \frac{\gamma}{1+\gamma} \Delta L. \quad (21)$$

The above argument shows that a photoresistor may be treated as a circuit element if we use the three basic parameters A_θ, α and γ.

A photoresistor is, as a rule, the circuit element that shows the largest temperature coefficient.

Photoresistor circuits have to be compensated against temperature changes at low light intensities. Figure 6 shows the relative change of current with temperature for some standard types. The curves show that the temperature coefficient can usually be taken as roughly constant in the range from 0 to 100°C.

Figure 5 then gives us that

$$R_H(t^\circ) = R_s(t^\circ) \frac{\beta_t}{1-\beta_t}, \quad (22)$$

where

$$\beta_t = \frac{R_H(t_0^\circ)}{R_H(t_0^\circ) + R_S(t_0^\circ)} \tag{23}$$

is given.

We take R_H to consist of a resistance R_0 of temperature coefficient α_R in series with, say, a thermistor of resistance R_T that has a temperature coefficient α_T. Then

$$R_H(t^\circ) = R_0(1 \pm \alpha_R \Delta t^\circ) + R_{T0}(1 \pm \alpha_T \Delta t^\circ)$$

and R_0 and R_{T_0} are given by

$$R_0 + R_{T0} = R_H(t_0^\circ), \tag{24}$$

$$R_0(1 \pm \alpha_R \Delta t^\circ) + R_{T0}(1 \pm \alpha_T \Delta t^\circ) = R_H(t^\circ). \tag{25}$$

Therefore,

$$R_0 = \frac{R_H(t_0^\circ)(1 \pm \alpha_T \Delta t^\circ) - R_H(t^\circ)}{\pm(\alpha_R + \alpha_T)}, \tag{26}$$

$$R_{T0} = \frac{R_H(t_0^\circ)(1 \pm \alpha_R \Delta t^\circ) - R_H(t^\circ)}{\pm(\alpha_T + \alpha_R)}. \tag{27}$$

Now $\beta_t = f(t^\circ)$, so the relative error in the compensation is

$$\beta_0 = \frac{\beta(t^\circ) - \beta(t_0^\circ)}{\beta(t_0^\circ)}, \tag{28}$$

Tests show that β_0 is usually 1-2% over the range 20-100°C. The transient response, or the frequency response, is controlled by the mean carrier lifetime, τ_0.

If the intensity-current curve is nonlinear, τ will be a function of intensity. This follows directly from (2) and (4). Now (4), (6) and (7) give us that

$$\alpha' = \alpha - 1.$$

Thus, if $\alpha < 1$, we have $\alpha' < 0$ and a rise with intensity in accordance with $\tau \sim \tau_0/L^{\alpha'}$. If $\alpha > 1$, we have $\alpha' > 0$, and τ falls as L rises.

The formal kinetic equation for $I_p(t)$ can be put as

$$\frac{dI_p}{dt} + \frac{1}{\tau} I_p = \beta^* L, \tag{29}$$

where $\beta^* = \beta\mu ekU/d^2$ is the quantum yield in current terms.

The solution to (29) also gives us the frequency response. In general, $\tau = \tau(L, I_p, t)$ and $\beta^* = \beta(L, I_p, t)$. The kinetics will clearly then be nonlinear. The linear case occurs when τ and β^* are constant. This, in fact, occurs only at high intensities, or if bias illumination is used. Any modulated beam has a certain mean illumination level L_0. Sinusoidal [6] and square-wave [7] modulations have been considered; the formulas for $I_p(t)$ are, respectively,

$$I_p = \frac{\beta^* \tau_0 L_0}{\sqrt{1 + 4\pi^2 f^2 \tau_0^2}} \tag{30}$$

and

$$I_p = \beta^* \tau_0 L_0 \phi h \left(\frac{1}{4f\tau_0}\right), \tag{31}$$

which also give the frequency response. Now τ_0 is the photoresistor parameter in (30) and (31). Table 2 gives the mean values of τ_0 for the standard types, and Fig. 7 gives the frequency-response curves recorded with square-wave modulation. The current $I_{po} = \beta^* \tau_0 L_0$ corresponds to f = 0.

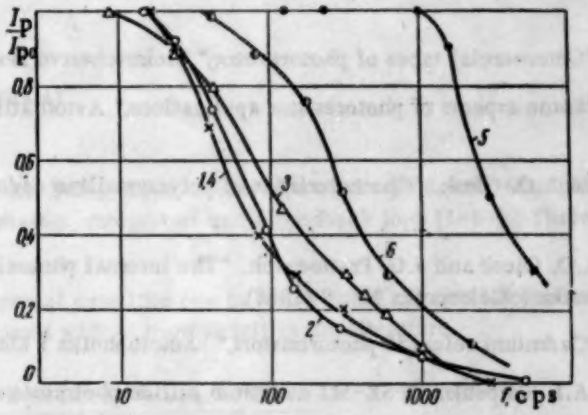

Fig. 7. Frequency-response curves for photoresistors: 1 and 4) FS-K1; 2) FS-K2; 3) FSK-M; 5) FS-A1; 6) FSS-M.

There is always a certain constant component in the illumination in such tests; this activates the specimen, and so we can assume that $\tau = \tau_0 = \text{const}$. This activation effect will appear only at high values of L if single flashes are used. At low intensities we have $\beta^* = \beta^*(t)$, as has been shown [4, 8, 9]. The initial section of the current curve is then hyperbolic. In fact, (29) shows that, when $t \ll \tau$,

$$I_p(t) = \beta^*(t) Lt. \tag{32}$$

We expand $\beta^*(t)$ in series form, and get a general expression for the initial section of the intensity response:

$$I_p(t) = \beta_0^* L t^n. \tag{33}$$

The special case $\beta^* = \text{const}$ corresponds to n = 1. If n > 1, the photoresistor shows a delay in its response; this delay will be the larger the smaller L.

This fact should be borne in mind in calculating photoresistor circuits, including correction circuits.

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THE STABILITY OF AN EXTRAPOLATION DEVICE

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Much attention is at present being given to one type of sampled-data extrapolator, namely to the type that contains a sampling unit, summators, integrators and a feedback loop [1-10]. These devices are useful in very many different ways.

This note deals with a special case (the one in which there are two integrators); it is, all the same, a very important case. It is the one dealt with in most detail in the literature.

1. Stability of an Extrapolator with a Gate

Consider the device shown in Fig. 1 (the symbols are the same as Tsypkin's, so far as is possible). The gate has $\gamma \ll 1$. The transfer function for the continuous-action part of the open-loop system is

$$Y(p) = \gamma T_r e^{-t_s p}\left(\frac{a}{p} + \frac{b}{T_r p^2}\right). \tag{1}$$

We introduce the relative time, and put $T_r p = q$, $\gamma a T_r = \alpha$, $\gamma b T_r = \beta$, $t_s/T_r = \tau$; we get

$$Y(q) = e^{-\tau q}\left(\frac{\alpha}{q} + \frac{\beta}{q^2}\right). \tag{2}$$

The original, corresponding to $Y(q)$, is

$$\begin{aligned} y(\bar{t}) &= 0 \qquad \text{when} \quad \bar{t} \leqslant \tau, \\ y(\bar{t}) &= \alpha + \beta(\bar{t} - \tau) \quad \text{when} \quad \bar{t} > \tau. \end{aligned} \tag{3}$$

Thus we put $e^q = z$, and get the pulse-transfer function for the open-loop system as

$$Y^*(z) = \frac{[\alpha + \beta(1-\tau)]z - \alpha + \beta\tau}{(z-1)^2} \tag{4}$$

the characteristic equation is

$$z^2 + [\alpha - 2 + \beta(1-\tau)]z + 1 - \alpha + \beta\tau = 0, \tag{5}$$

The stability conditions for the closed-loop system are

$$\begin{aligned} &\beta > 0, \\ &\alpha > \beta\tau \\ &\alpha < 2 - \beta\left(\frac{1}{2} - \tau\right). \end{aligned} \tag{6}$$

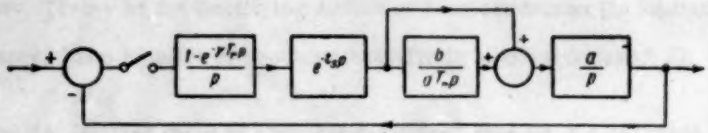

Fig. 1.

The characteristic equation and the stability conditions are derived in another way in [5].•

The special case $\tau = 0$ has been dealt with [9], where the stability region is split up into subregions that correspond to different forms for the roots of the characteristic equation.

Theorem 1. Let the delay time τ vary over the range $0 \leq \tau < 1$. Then the triangular stability region (that found for $\tau = 0$) is deformed (Fig. 2) in such a way that all the ordinates β, on lines that split the triangle into subregions, remain unchanged for any $\tau \neq 0$; the abscissas α are displaced linearly by $\beta\tau$.

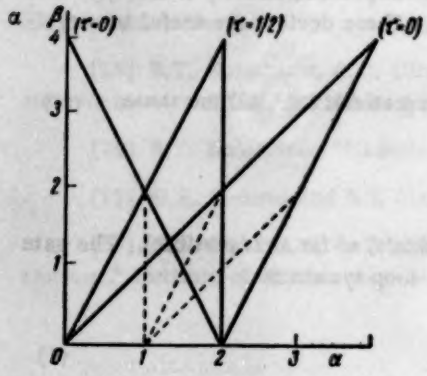

Fig. 2.

The proof of this theorem follows from (5). Clearly, the point (α, β) for $\tau = 0$ and the point $(\alpha + \beta T, \beta)$ for $\tau = T$ correspond to identical roots.

Theorem 2. The following inequalities give the bounds to a subregion, to which bounds correspond two real roots of the characteristic equation (one positive, one negative):

$$\begin{aligned} &\beta > 0, \\ &\alpha > 1 + \beta\tau, \\ &\alpha < 2 - \beta\left(\frac{1}{2} - \tau\right). \end{aligned} \tag{7}$$

The proof of this theorem follows from (5). The necessary and sufficient condition for two real roots of opposite signs to exist is that the following inequalities are all complied with at once:

$$\begin{aligned} &[\alpha + \beta(1-\tau)]^2 > 4\beta, \\ &1 - \frac{\alpha + \beta(1-\tau)}{2} + \sqrt{\left(\frac{\alpha + \beta(1-\tau)}{2}\right)^2 - \beta} > 0, \\ &1 - \frac{\alpha + \beta(1-\tau)}{2} + \sqrt{\left(\frac{\alpha + \beta(1-\tau)}{2}\right)^2 - \beta} < 0. \end{aligned} \tag{8}$$

It is readily seen that the second and third conditions in (8) are complied with at once only if

$$\alpha > 1 + \beta\tau. \tag{9}$$

But the first condition in (8) is then also complied with. We add the first and third conditions in (6), and so get (7).

Figure 2 shows three special cases, namely those for $\tau = 0$, $\tau = 1/2$, $\tau = 1$ (see also [5]). The full lines correspond to the conditions of (6), and the broken lines to the second condition in (7).

2. Stability of an Extrapolator with a Clamp

In this case $\gamma = 1$. We consider only the special case $\tau = 0$. Then

•There are misprints in [5]; the term βz in the characteristic equation has a minus sign, and the third stability condition should have < instead of >.

$$Y(p)=\frac{1-e^{-T_r p}}{p}\left(\frac{a}{p}+\frac{b}{T_r p^2}\right). \tag{10}$$

We put $aT_r = \alpha$, $bT_r = \beta$, and get

$$Y(q)=\frac{1-e^{-q}}{q}\left(\frac{\alpha}{q}+\frac{\beta}{q^2}\right). \tag{11}$$

Thus we put $e^q = z$, and get the pulse-transfer function for the open-loop system as

$$Y^*(z)=\frac{\left[\alpha+\frac{1}{2}\beta\right]z-\alpha+\frac{1}{2}\beta}{(z-1)^2}. \tag{11}$$

Theorem 3. The circuit with a clamp with $\tau = 0$ is equivalent to the one with the gate when $\tau = 1/2$.

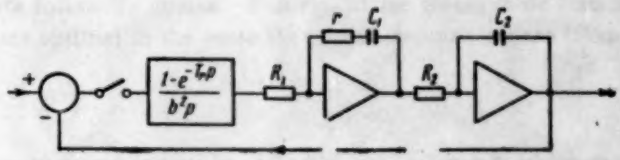

Fig. 3.

The proof of this follows from (12) and (4) taken with $\tau = 1/2$.

The clamp circuit (Fig. 3) has been considered in detail [2, 10]. In [10] use is made of the parameters

$$j=\frac{b}{T_r}\sqrt{R_1 R_2 C_1 C_2} \quad \text{and} \quad k=\frac{rC_1}{T_r} \tag{13}$$

and the bifurcation of the stability region is given for these. Now α and β are related to j and k:

$$j=\frac{1}{\sqrt{\beta}}, \quad k=\frac{\alpha}{\beta}, \qquad \alpha=\frac{k}{j^2}, \quad \beta=\frac{1}{j^2}. \tag{14}$$

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A PEAK-HOLDING REGULATOR THAT FOLLOWS THE PEAK

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(Moscow)

A peak-holding regulator that follows the peak is described. It consists of two main parts, one of which measures the distance from the peak, and the other of which is a sampled-data follow-up device. Test results are given; these tests had as object to find parameters optimal in the sense that the root-mean-square follow-up error is minimal.

The extremum (maximum or minimum) has to be sought in cases in which the peak changes its position continuously. The response of the system is improved if a servo is used to follow the extremum.

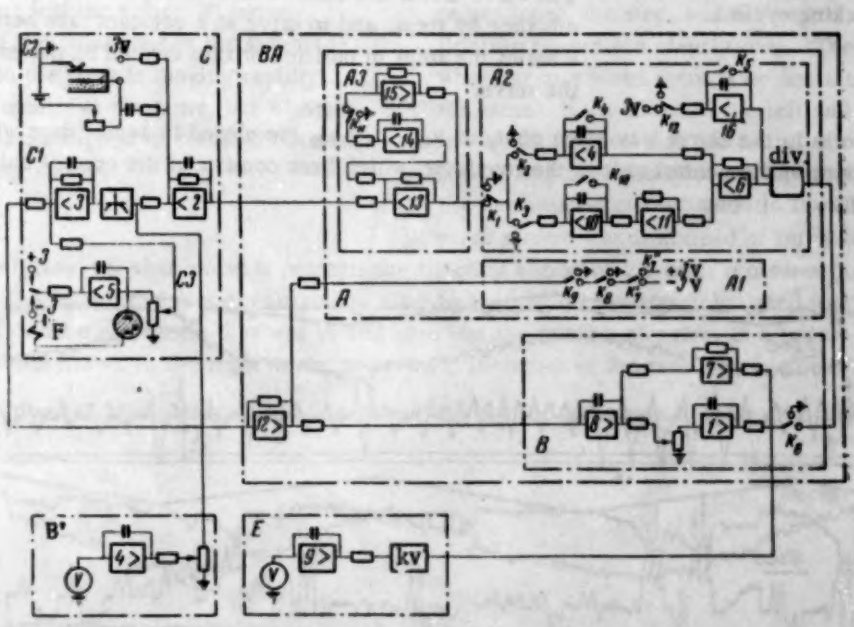

Fig. 1. The experimental system for studying a peak-holding regulator that follows the peak.

The author's system, which is described here, used the signal that indicates the deviation from the peak as the error signal for the sampled-data servo. A regulator of this type is under development for industrial purposes; a pneumatic system is to be used. An electronic analog was first tested.* The controlled object and the device for adjusting the regulator to the peak were included in the analog. The tests showed that the system had some advantages over other peak-holding devices that have been described.

*From proceedings of the experimental seminar on pneumo- and hydroautomation control, IAT, AN SSSR, May 22, 1958.

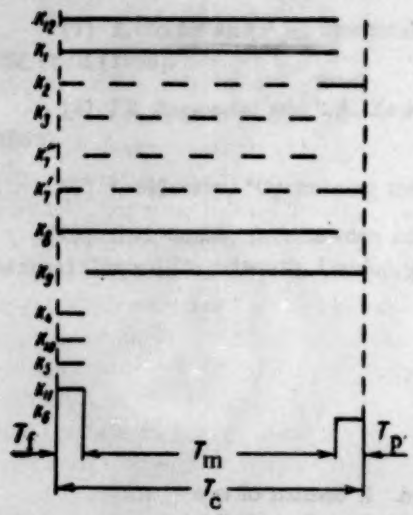

Fig. 2. Diagram of the working sequence of the contacts in the control unit. T_m is the time taken to measure the distance from the peak, T_p is the pulse length in the servo, T_f is the time for which the floating-zero system operates, and during which the integrators in A2 discharge, and T_c is the time for one working cycle.

1. The electronic regulator. This (Fig. 1, BA) has three main parts, namely the mismatch measurer (A), the sampled-data servo (B) and the control unit (not shown).

Unit A consists of an oscillation source (A1), the measurer itself (A2), and a floating-zero read-off system (A3).

Unit A1 of Fig. 1 supplies a train of square pulses to the input of the controlled object. The pulse length and train length can be varied. The output signal produced from the object by these pulses gives the magnitude and sign of the derivative of the steady-state characteristic for the point at which the object actually is. This output is fed to two integrators (4 and 10 in Fig. 1). One integrator deals with the output when positive inputs are supplied to the object, the other with the output corresponding to negative inputs. The outputs from the integrators are differenced, for which purpose an inverter and summator are used (amplifiers 11 and 6 in Fig. 1). The resulting output signal (Fig. 1,A and Fig. 3, curve 4) specifies the distance from the peak. This method, as is usual, reduces the effects of noise. The fixed component in the output from the object is balanced out up to the start of the measurement by connecting in the floating system by closing contact k_{11}. At the same time, contacts k_4 and k_{10} are closed to discharge the integrators 4 and 10.

The resulting output signal is fed to the servo by closing k_6. The servo drives the controller in two distinct ways; it causes it to advance by steps, and to drive at a constant rate between steps. The sizes of the steps in position or rate depend on the error signal fed to the servo.

The system works in the above way when contacts $k_1, \ldots, k_n$ are closed in accordance with Fig. 2, which describes the operations of the control unit in the regulator, which here consists of the control unit of the electronic analog and of a set of contactors.

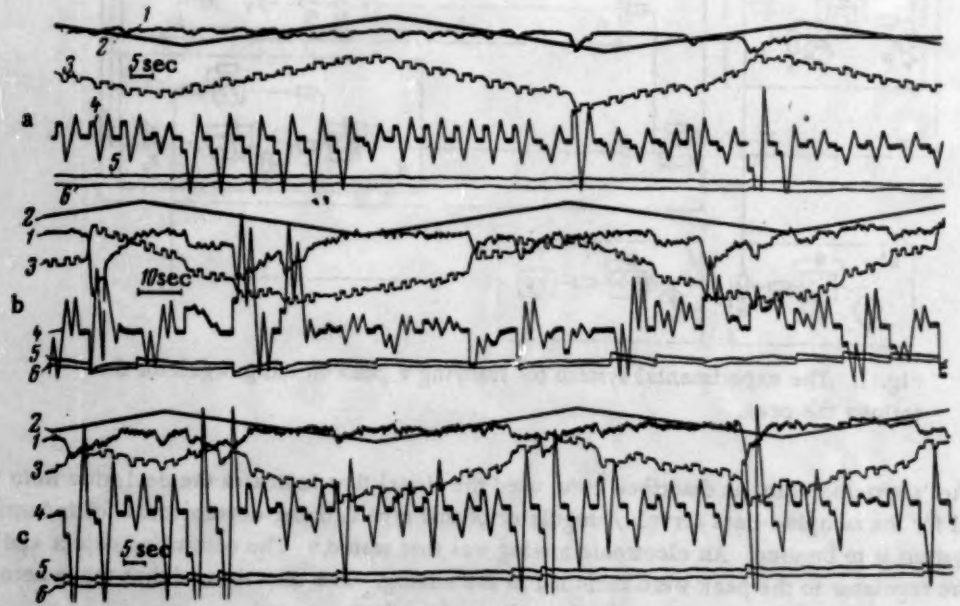

Fig. 3. 1) Output from object; 2) position of controller corresponding to the extreme value of the output; 3) position of controller during follow-up; 4) distance from peak; 5 and 6) outputs from integrators 4 and 10.

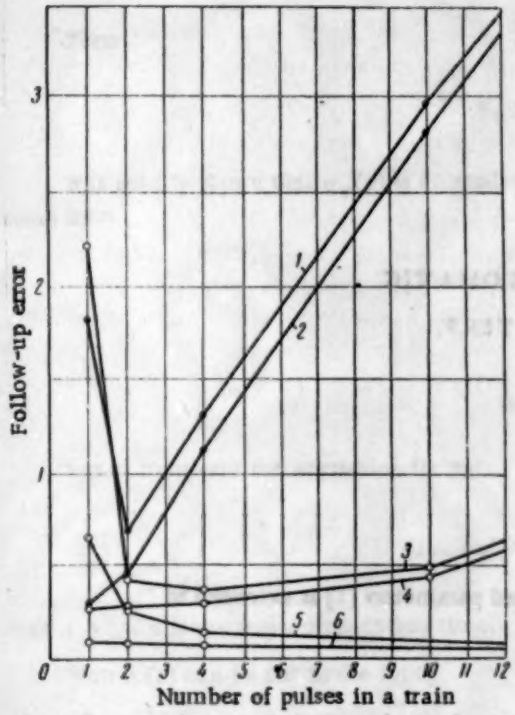

Fig. 4. 1) Root-mean-square follow-up error when the peak is moving and noise is present; 2) the same, but without noise; 3) mean-square value of the pulses fed to the sampled-data servo when the peak is moving rapidly and noise is present; 4) the same, but without noise; 5 and 6) correspond to 3 and 4, but with the peak moving slowly.

2. Adjusting the regulator. The parameters of the pulse train can be adjusted; so can the position and rate steps produced by the servo in the controller. These adjustments are made manually to give minimum follow-up error under given conditions. The optimum parameters were selected by measuring the root-mean-square (integral) error. Unit B' in Fig. 1 was used for this purpose. The actual position of the peak is, of course, unknown under real conditions; therefore, unit E (Fig. 1) measured the mean value of the square of the pulse input to the servo, which is a quantity that can be found in practice. There would, in principle, be no difficulty in making the adjustments automatically. The peak-holding regulator could be adjusted continuously by an analogous peak-holding regulator, which would seek out those values of the parameters for first regulator that result in the minimum mean-square input to the servo.

3. Results. The analog to the object (Fig. 1,C) was a squaring unit with lags before and after it (C1), a ball noise generator (C2) and a source (C3) that produced sawtooth oscillations in the position of the peak without affecting the peak height. The parameters of these various units could be altered. Figure 3a shows the results for zero noise. Curve 3 shows the motion of the controller; the train of pulses (one), the step, and the period of operation of the floating system are clearly seen. The left section shows what happens when there is no continuous motion between the steps. If we compare the left and right halves of curve 1 in Fig. 3a, we see that the continuous motion of the controller improves the response; the follow-up worsens around the extrema on curve 2 of Fig. 3, which shows that the servo system stands in need of improvement. Figure 3b and c show the follow-up when noise is present; the filtering action of the error measurer may be judged from these curves. The relation of root-mean-square error to the parameters was calculated from the results. Figure 4 shows an example (the parameter that was varied here was the number of pulses in a train); there is a marked optimum point which moves to the right as the noise level increases or the rate of movement falls.

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METHODS OF STUDYING THE STABILITY OF AUTOMATIC CONTROL SYSTEMS WITH DISTRIBUTED PARAMETERS, WITH ALLOWANCE FOR LOSSES

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(Baku)

The stability criterion for systems with distributed parameters [1] is extended to systems with losses.

Consider automatic control systems with distributed parameters whose characteristic equations can be put in the form

$$\psi_1(p)\,e^{\frac{1}{2}\gamma l} - \psi_2(p)\,e^{-\frac{1}{2}\gamma l} = 0, \tag{1}$$

where $\psi_1(p)$, $\psi_2(p)$ are polynomials of degrees $\underline{n}$ and $\underline{m}$, with $m \leq n$,

$$\gamma = \sqrt{p^2 c + pb}, \tag{2}$$

or, in the general case,

$$\gamma = \sqrt{p^2 c + pb + a}, \tag{3}$$

where l, $\underline{c}$, $\underline{b}$ and $\underline{a}$ are the parameters of the part with distributed constants.

Consider the function

$$\varphi_\gamma(p) = \frac{\psi_1(p)\,e^{\frac{1}{2}\gamma l} - \psi_2(p)\,e^{-\frac{1}{2}\gamma l}}{\psi_1(p)\,e^{\frac{1}{2}\gamma l}} = 1 - \frac{\psi_2(p)}{\psi_1(p)}\,e^{-\gamma l}, \tag{4}$$

where zeros equal the roots of (1), and whose poles equal the roots of

$$\psi_1(p) = 0. \tag{5}$$

We use the notation

$$\frac{\psi_1(p)}{\psi_2(p)} = W(p). \tag{6}$$

Then

$$\varphi_\tau(p) = 1 - \frac{1}{W(p)}\, e^{-\gamma l}. \tag{7}$$

It is readily shown that $\varphi_\tau(p)$ is an analytic function. Then the critical values of l_{cl} and ω_{cl} can [2] be found from

$$\varphi_\tau(j\omega) = 1 - \frac{1}{W(j\omega)}\, e^{-j\sqrt{\omega^2 c - j\omega b}\, l} = 0 \tag{8}$$

or

$$W(j\omega) = e^{-j\sqrt{\omega^2 c - j\omega b}\, l}. \tag{9}$$

Let us transform the expression for γl:

$$\gamma l = l\sqrt{p^2 c + pb} = \tau\sqrt{p^2 + p\frac{b}{c}} = \lambda(p). \tag{10}$$

Here $\tau = l/\sqrt{c}$ is the wave propagation time.

Now $\lambda(p)$ can be put in the form

$$\lambda(p) = \tau p\sqrt{1 + \frac{b}{cp}}. \tag{11}$$

We expand $\sqrt{(1 + b/cp)}$ as a series [3] and get

$$\lambda(p) = \tau p\left[1 + \frac{1}{2}\frac{b}{c}\frac{1}{p} - \frac{1}{2\times 4}\left(\frac{b}{c}\right)^2\frac{1}{p^2} + \frac{1\times 3}{2\times 4\times 6}\left(\frac{b}{c}\right)^3\frac{1}{p^3} - \cdots\right]. \tag{12}$$

The series converges if $|1/p| \leq 1$. This condition is obeyed with $p = j\omega$ if $|\omega| > 1$.

Thus, if $|\omega| > 1$, we have

$$\lambda(j\omega) = \tau\left[j\omega + \frac{1}{2}\frac{b}{c} + j\frac{1}{2\times 4}\left(\frac{b}{c}\right)^2\frac{1}{\omega} - \frac{1\times 3}{2\times 4\times 6}\left(\frac{b}{c}\right)^3\frac{1}{\omega^3} - \cdots\right]. \tag{13}$$

But $\lambda(p)$ can be put in another form

$$\lambda(p) = \tau\sqrt{p\frac{b}{c}}\sqrt{1 + \frac{c}{b}p}.$$

We expand $\sqrt{(1 + cp/b)}$ and get

$$\lambda(p) = \tau\sqrt{p\frac{b}{c}}\left[1 + \frac{1}{2}\frac{c}{b}p - \frac{1}{2\times 4}\left(\frac{c}{b}\right)^2 p^2 + \frac{1\times 3}{2\times 4\times 6}\left(\frac{c}{b}\right)^3 p^3 - \cdots\right]. \tag{14}$$

This series converges if $|p| \leq 1$, i.e., with $p = j\omega$ the expansion applies if $|\omega| < 1$. Now $\sqrt{1} = \sqrt{2}/2(1+j)$, so if $|\omega| < 1$, we have

$$\lambda(j\omega) = \tau\sqrt{\frac{b}{c}\omega}\frac{\sqrt{2}}{2}(1+j)\left[1 + \frac{1}{2}\frac{c}{b}j\omega + \frac{1}{2\times 4}\left(\frac{c}{b}\right)^2\omega^2 - \cdots\right]. \tag{15}$$

We need take only the first two terms in (13) and (15). If $|\omega| > 1$, we use the following expression for $\lambda(j\omega)$:

$$\lambda(j\omega) = \tau\left(j\omega + \frac{1}{2}\frac{b}{c}\right). \tag{16}$$

Then (11) and (16) give

$$W(j\omega) = e^{-\frac{1}{2}\frac{b}{c}\tau} e^{-j\omega\tau} = e^{-m\tau} e^{-j\omega\tau}, \tag{17}$$

where

$$m = \frac{1}{2}\frac{b}{c}.$$

Hence W(jω) can be put as

$$W(j\omega) = |W(j\omega)|\, e^{j \operatorname{arc\,tg} f(\omega)}, \tag{18}$$

where $|W(j\omega)|$ is the modulus of W(j,ω), and arc tg f(ω) is the argument of W(j,ω).

We substitute (18) into (17) and get

$$|W(j\omega)|\, e^{j \operatorname{arc\,tg} f(\omega)} = e^{-m\tau} e^{-j\omega\tau}. \tag{19}$$

Taking logarithms of both parts of (19), we have

$$\ln|W(j\omega)| + j \operatorname{arc\,tg} f(\omega) = -m\tau - j\omega\tau. \tag{20}$$

Separating the real and imaginary parts, we have

$$\ln|W(j\omega)| = -m\tau, \tag{21}$$

$$\operatorname{arc\,tg} f(\omega) = -\omega\tau, \tag{22}$$

or

$$\tau = -\frac{1}{m}\ln|W(j\omega)|, \tag{23}$$

$$\tau = -\frac{1}{\omega}\operatorname{arc\,tg} f(\omega). \tag{24}$$

If $|\omega| < 1$, we will have

$$\lambda(j\omega) = \sqrt{m\omega}\,(1+j)\left(1 + \frac{1}{4m}j\omega\right). \tag{25}$$

We perform the same expansions as for $|\omega| > 1$, and get for $|\omega| < 1$ that

$$\tau = -\frac{4m}{(4m-\omega)\sqrt{m\omega}}\ln|W(j\omega)|, \tag{26}$$

$$\tau = -\frac{4m}{(4m+\omega)\sqrt{m\omega}}\operatorname{arc\,tg} f(\omega). \tag{27}$$

We give ω values from $-\infty$ to ∞, and construct the $\tau = f_1(\omega)$ curve from (23) and (26), and the $\tau = f_2(\omega)$ curve from (24) and (27). The points where these curves meet define ω_{cri} and τ_{cri}.

We define the stability region from the limiting characteristic (for $\tau = 0$) in the way given in [1, 2].

We now demonstrate how to find ω_{cri} and τ_{cri} without resort to expanding $\lambda(p) = \gamma L$ in series form.

We put $\sqrt{\omega^2 - j\omega\frac{b}{c}}$ in the form

$$\sqrt{\omega^2 - j\omega\frac{b}{c}} = \sqrt[4]{\omega^4 + \left(\frac{b}{c}\right)^2\omega^2}\, e^{-j0.5 \operatorname{arc\,tg}\frac{b}{c\omega}}. \tag{28}$$

We replace (11) by

$$|W(j\omega)|\, e^{j \operatorname{arc\,tg} f(\omega)} = e^{-j\tau\sqrt[4]{\omega^4 + \left(\frac{b}{c}\right)^2\omega^2}\, e^{-j0.5 \operatorname{arc\,tg}\frac{b}{c\omega}}} \tag{29}$$

Taking logarithms of both parts of (29), we have

$$\ln|W(j\omega)| + j \operatorname{arc\,tg} f(\omega) = -j\tau\sqrt[4]{\omega^4 + \left(\frac{b}{c}\right)^2\omega^2}\, e^{-j0.5 \operatorname{arc\,tg}\frac{b}{c\omega}} \tag{30}$$

or

$$-j\tau\sqrt[4]{\omega^4 + \left(\frac{b}{c}\right)^2\omega^2} = e^{j0.5 \operatorname{arc\,tg}\frac{b}{c\omega}}\{\ln|W(j\omega)| + j \operatorname{arc\,tg} f(\omega)\}. \tag{31}$$

We give ω values from $-\infty$ to ∞, and construct the curve

$$\xi(j\omega) = e^{j0.5 \operatorname{arc\,tg}\frac{b}{c\omega}}\{\ln|W(j\omega)| + j \operatorname{arc\,tg} f(\omega)\}. \tag{32}$$

The points where $\xi(j\omega)$ meets the imaginary axis define ω_{cri} and τ_{cri}.

The regions of stability and instability are then derived as above.

The same method can be used to study the stability of systems that have (3) as their expressions for γ.

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